Introduction to the SIR model

MATHEMATICAL MODELING FOR INFECTIOUS DISEASE PLANNING IN AFRICA

(30 minutes)

UNIVERSITY OF Global Health EQUITY
HARVARD T.H. CHAN SCHOOL OF PUBLIC HEALTH
CENTER for COMMUNICABLE DISEASE DYNAMICS

Learning objectives

1. Understand how to design an SIR model and incorporate parameters according to disease epidemiology
2. Understand how to interpret and extract insights from basic outputs of an SIR model

Summary of the health states*

- **S**: Number of **S**usceptible individuals. If in contact with an infectious person, a susceptible becomes *infectious* depending on the model, and transitions into the infectious compartment.

- **I**: Number of **I**nfectious individuals. Infectious individuals can infect the susceptible population.

- **R**: Number of **R**ecovered individuals. These individuals have recovered from the infection and cannot be infected again (they are immune).

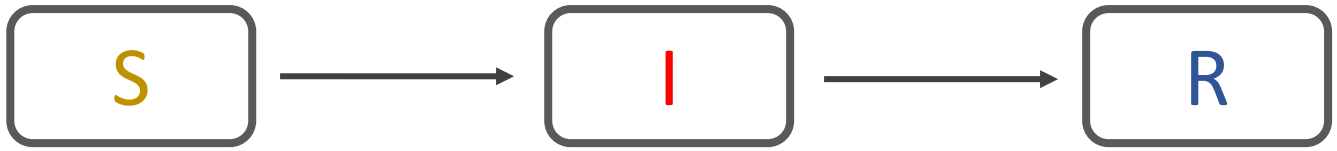
*Adapted from the "Infectious Disease Modeling in the Time of COVID" course notes by Flavia Camponovo, PhD and Caroline Buckee, DPhil

Transmission activity

- Form three groups. Each group will suggest definitions
  - Susceptible (S) group: 7 members
  - Infectious (I) group: 3 members
  - Recovered (R) group: No member

- Perform a few rounds of inter-team movements. At each round,
  1. Simulate infection (S -> I)
  2. Simulate recovery (I -> R)

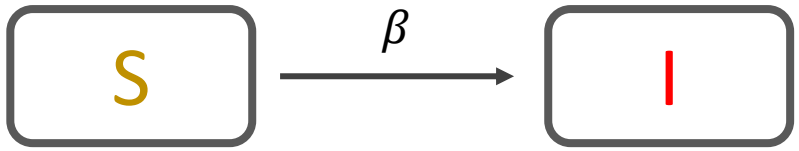
Structure of basic SIR model



- Team exercise: Assume
  - a recovery rate of γ per day,
  - a transmission rate of β per day,
  - A population size of N individuals

- Write down equations to describe how S, I and R will change with respect to time, t.

Structure of basic SIR model



- Model equations

$$\frac{dS}{dt} = \frac{-\beta SI}{N},$$

$$\frac{dI}{dt} = \frac{\beta SI}{N}$$

$$N = S + I$$

What insights could we gain from the SI model?

Scenario:
There are reports of an outbreak of a new emerging infectious disease that is transmitted from human to human. Not much is known about it, but we have been asked to create some initial models to analyze its spread. Based on initial reports, it appears:

- People have ~20% daily probability of getting infected upon contact with infected individuals
- Therefore, $\beta = ?$

After the introduction of an infected individual in a population of 10,000 individuals, how does the number of infected individuals change over time?

Dynamics of SI model

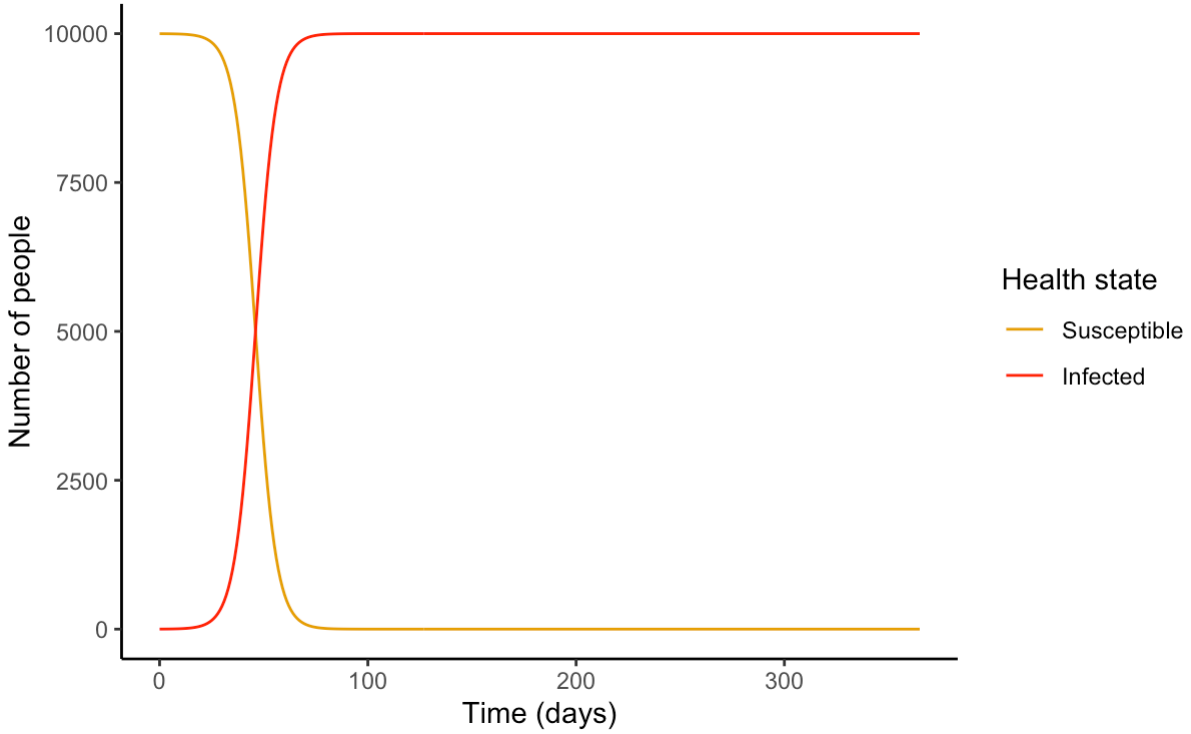


Figure: Dynamics of the number of susceptible(S), infectious (I) and recovered (R) individuals in a population of 10,000 after the introduction of one infected individual. $S(0) = 9999,\ I(0) = 1,\ \beta = 0.2,$

Structure of basic SIR model

$$\boxed{S} \xrightarrow{\beta} \boxed{I} \xrightarrow{\gamma} \boxed{R}$$

- Model equations

$$\frac{dS}{dt} = \frac{-\beta SI}{N},$$

$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I,$$

$$\frac{dR}{dt} = \gamma I,$$

$$N = S + I + R.$$

What insights could we gain from the SIR model?

Scenario:
There are reports of an outbreak of a new emerging infectious disease that is transmitted from human to human. Not much is known about it, but we've been asked to create some initial models to analyze its spread. Based on initial reports, it appears:

- People have ~20% daily probability of getting infected upon contact with infected individuals
- People recover on average 10 days after infection
- Therefore, $\beta = ?, \gamma = ?$

After the introduction of an infected individual in a population of 10,000 individuals, how does the number of infected individuals change over time?

What insights could we gain from the SIR model?

- *How long did the outbreak last?*
- *What fraction of the population was not affected?*
- *On what day (approximately) does the outbreak peak?*

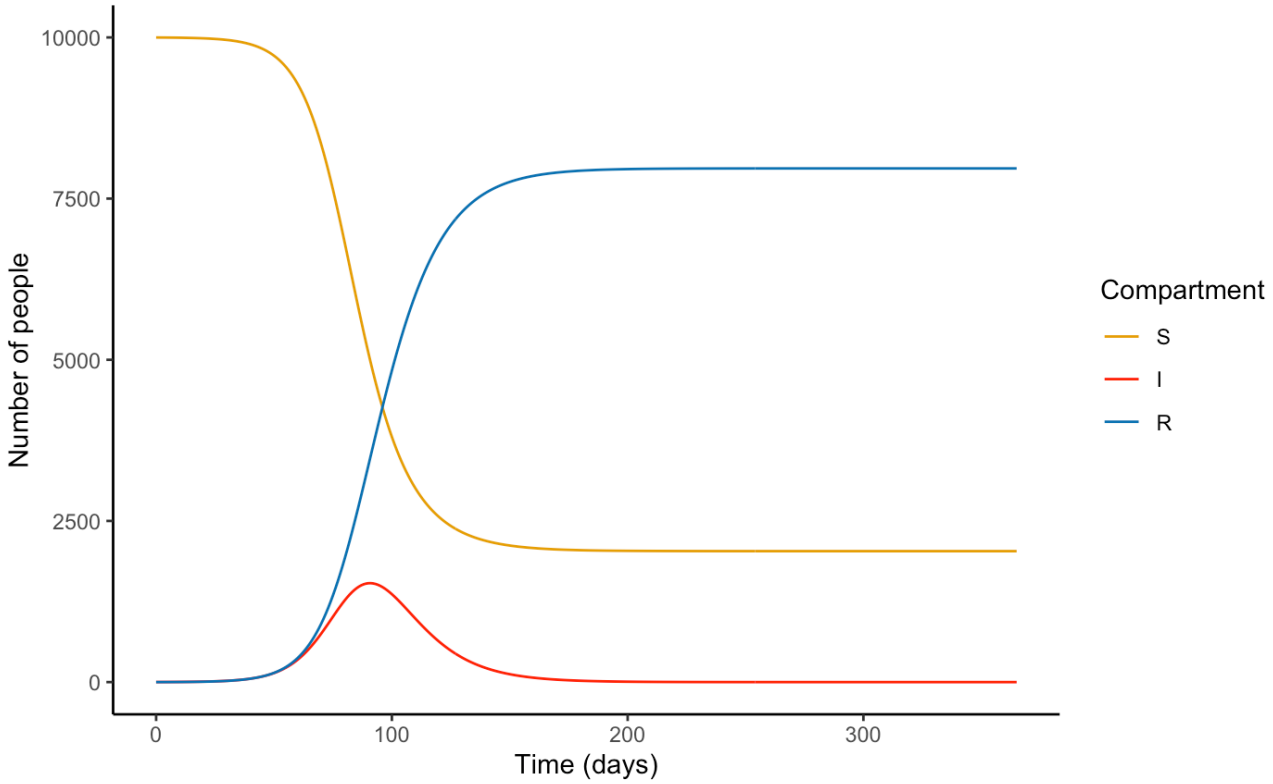


Figure: Dynamics of the number of susceptible(S), infectious (I) and recovered (R) individuals in a population of 10,000 after the introduction of one infected individual. $S(0) = 9999$, $I(0) = 1,\ R(0) = 0,\ \beta = 0.2,\ \gamma = 1/10$.

How do dynamics change with changing parameter values?

- Demonstration in Berkeley Madonna

What's coming next?

- **Activity**: Opportunity to apply concepts learnt to a case of a measles outbreak

- **Later lectures**: extensions to the SIR model, evaluating health outcomes and interventions, etc.