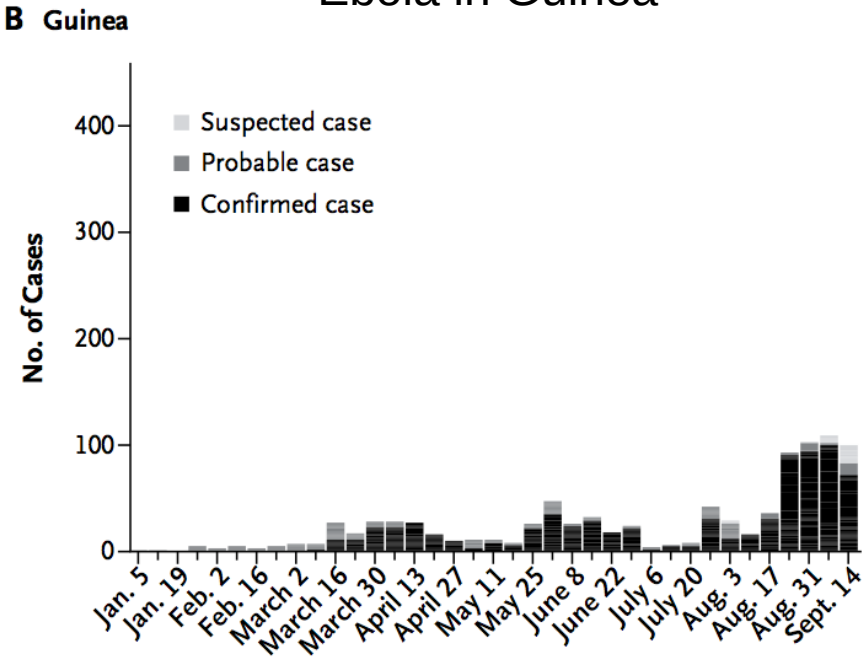
Introduction to basic reproduction numbers

MATHEMATICAL MODELING FOR INFECTIOUS DISEASE PLANNING IN AFRICA

(45 minutes)

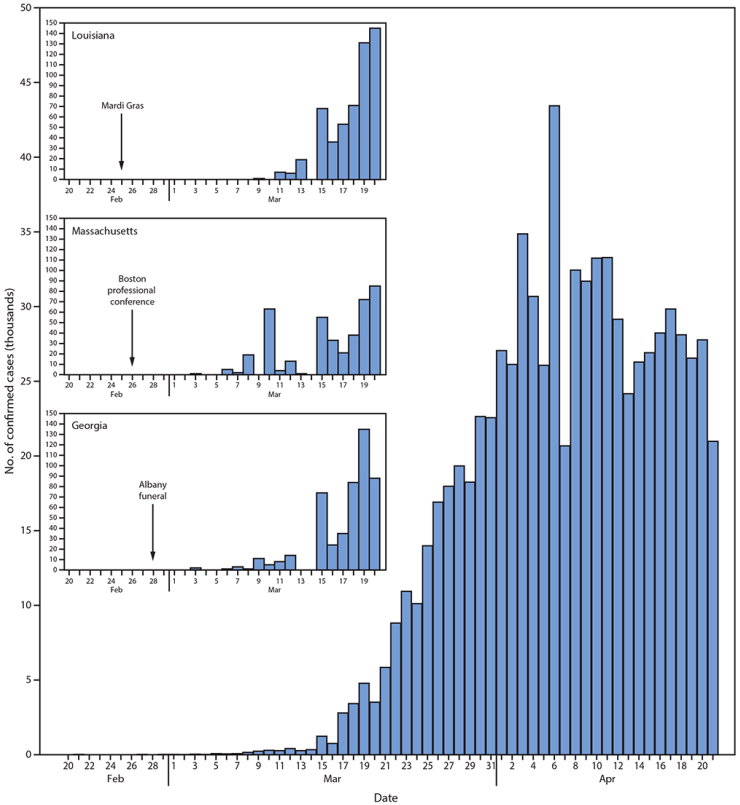
Ebola in Guinea



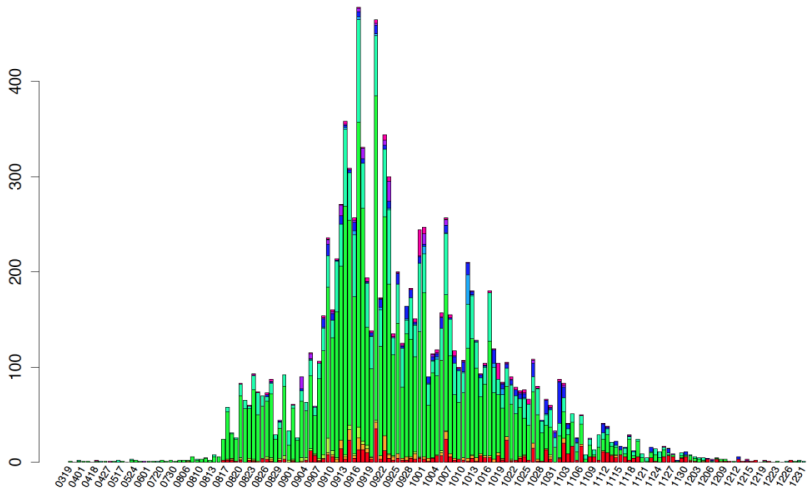
The Economist

Stochastic effects driven by introduction events and sputtering chains of transmission

Exponential growth phase “deterministic”



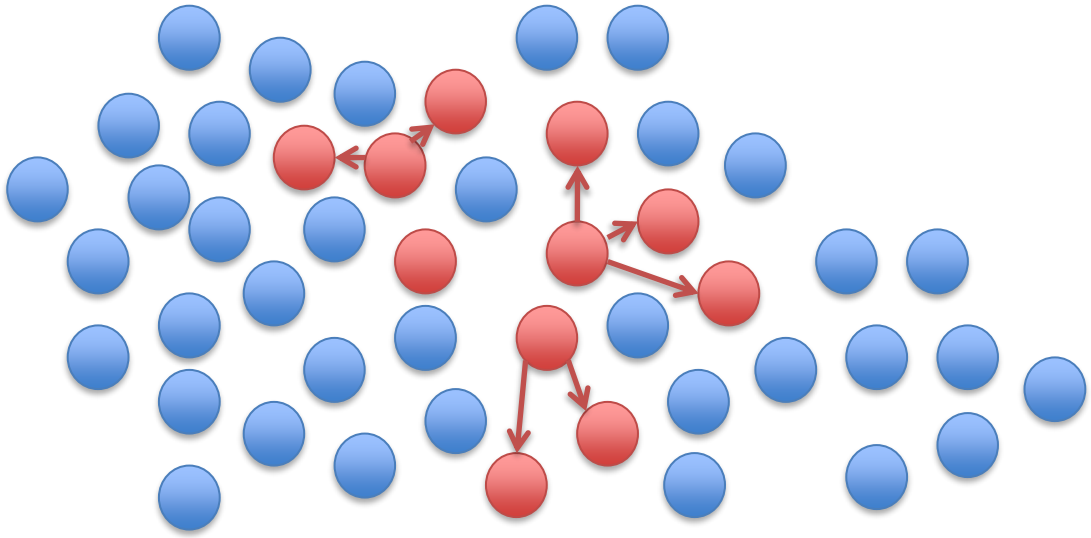
COVID-19



Dengue

The basic reproductive number R0

The number of people who develop an infectious disease as the result of infection by a single infectious case introduced into an entirely susceptible population.



R_0 is a useful threshold concept

- If $R_0 > 1$, disease is epidemic
- If $R_0 = 1$, disease is endemic
- If $R_0 < 1$, disease will die out

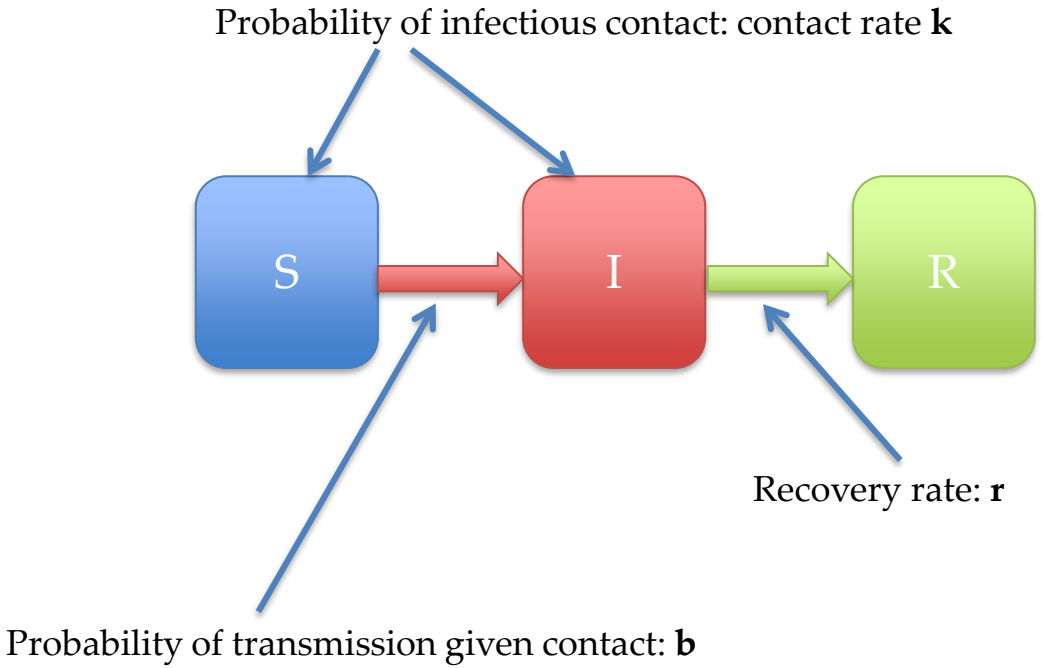
Basic Reproduction Number R0

R0 > 1: disease persistence

R0 < 1: disease extinction

R0 = ratio of people becoming infected to people recovering = **β/r**

Threshold condition to eliminate a disease



$$R0 = bk/r$$

Lower case letters are proportions (S/N, I/N, R/N)

$$\frac{ds}{dt} = -\beta si$$

$$\boxed{\frac{di}{dt} = \beta si - \nu i}$$

$$\frac{dr}{dt} = \nu i$$

Epidemic will occur when the **rate of change** of infected people is > 0

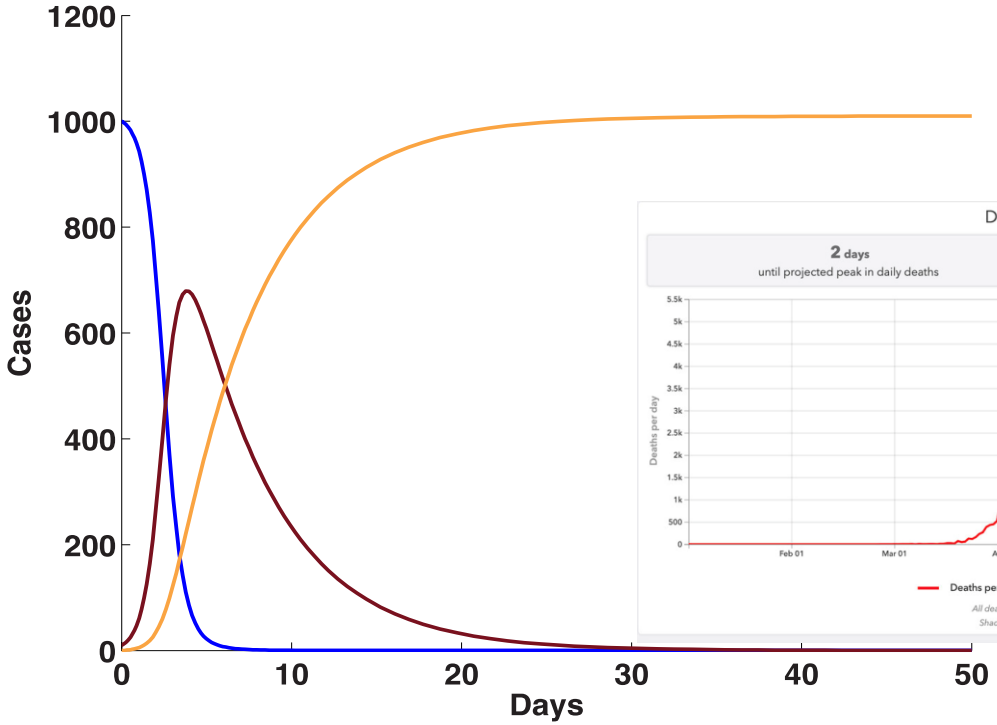
$$\beta si - \nu i > 0$$

At the start of an epidemic, s is more or less 1

$$\frac{\beta \boxed{s} i}{\nu} > i$$

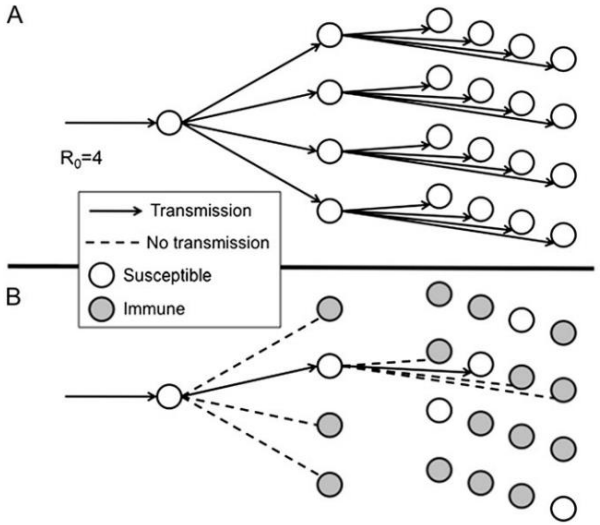
$$\frac{\beta}{\nu} = \mathcal{R}_0 > 1$$

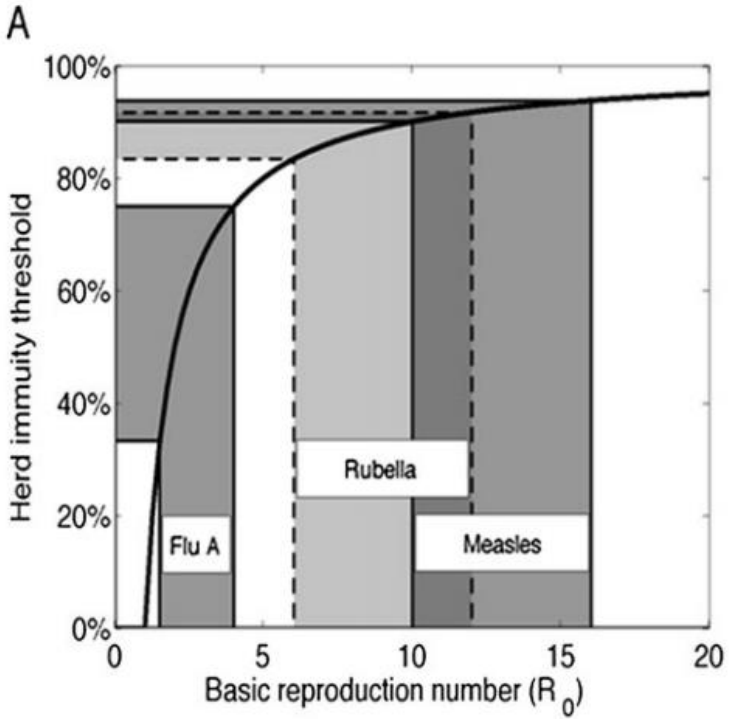
General findings of SIR models



Epidemics die out because

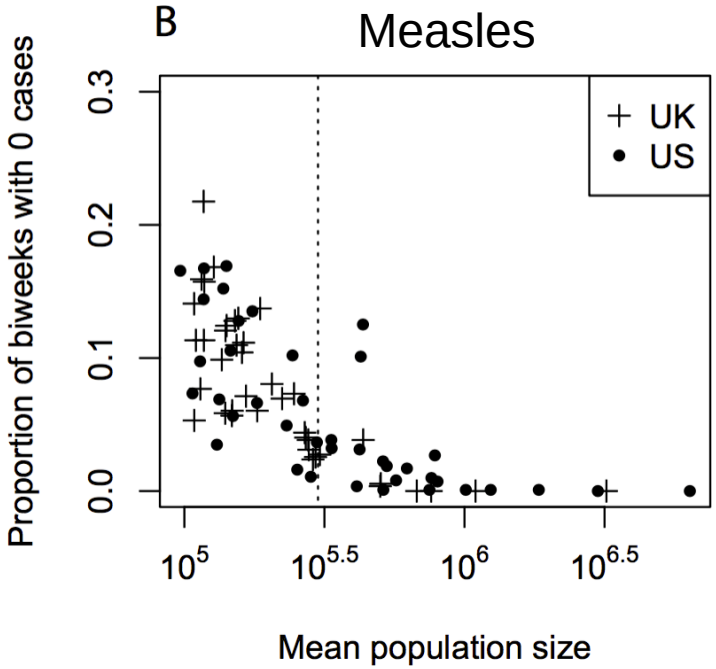
General findings of SIR models: herd protection

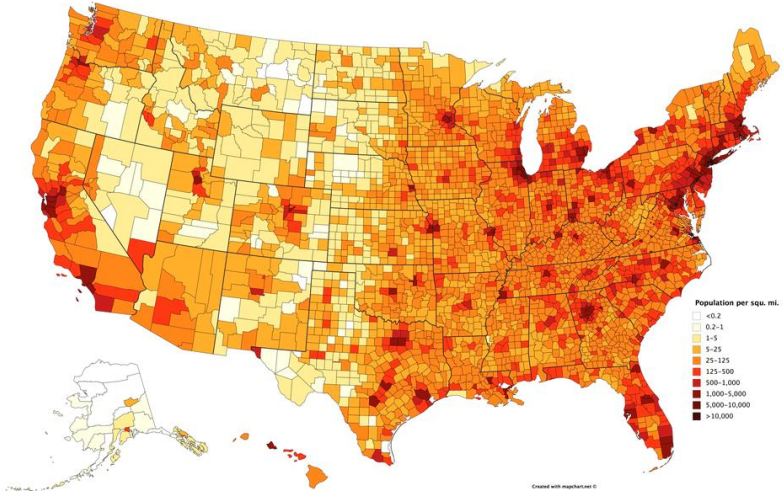




Fine et al (2011) Vaccines

General findings of SIR models: critical community size

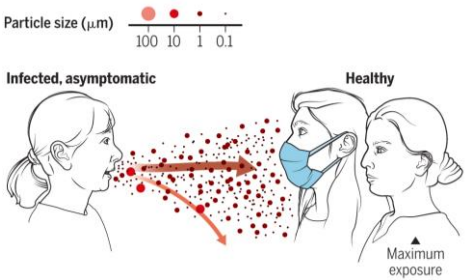


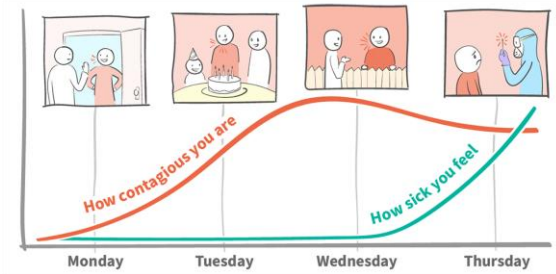


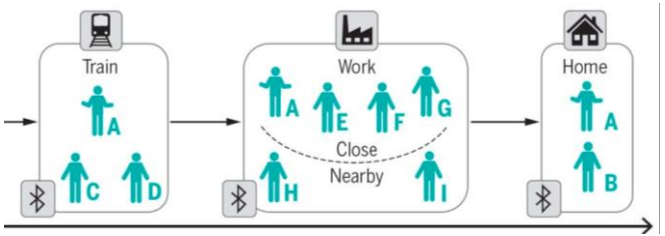
Dalziel (2016), but really Ba

The basic reproduction number

Probability of infection given contact (contagiousness) X Duration of infectiousness X Contacts per unit time





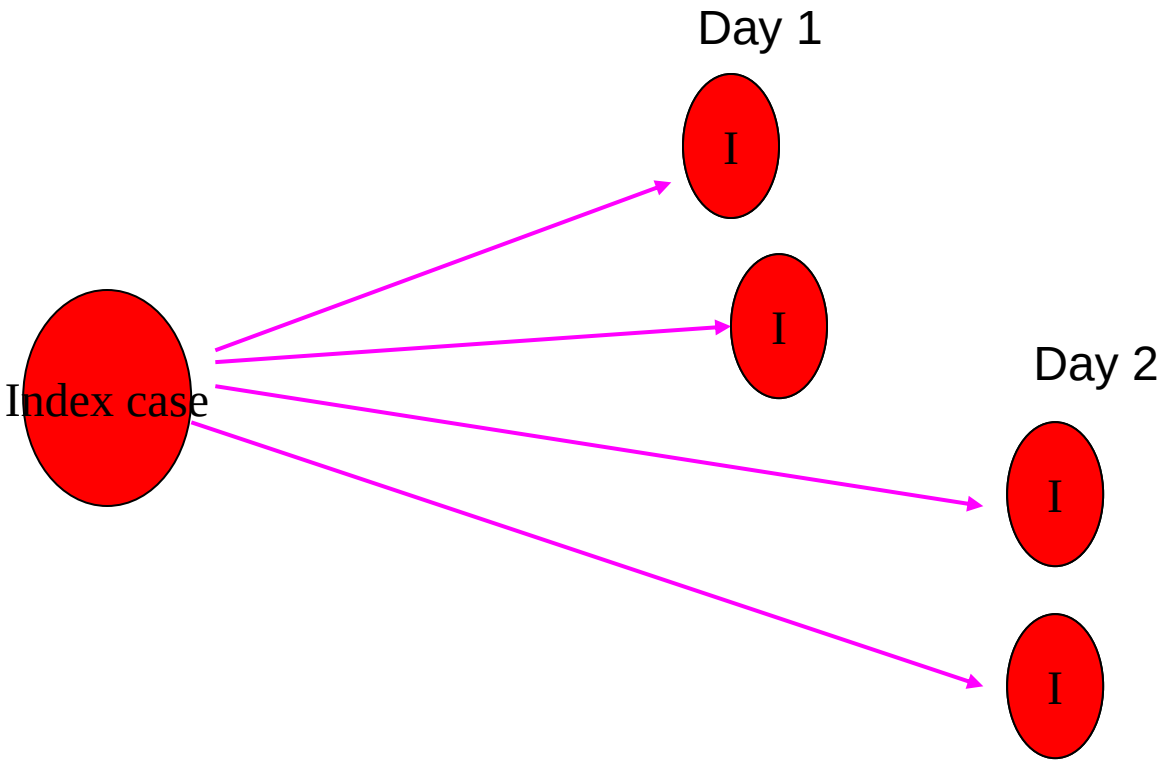


MASKS

TRACE/ISOLATE

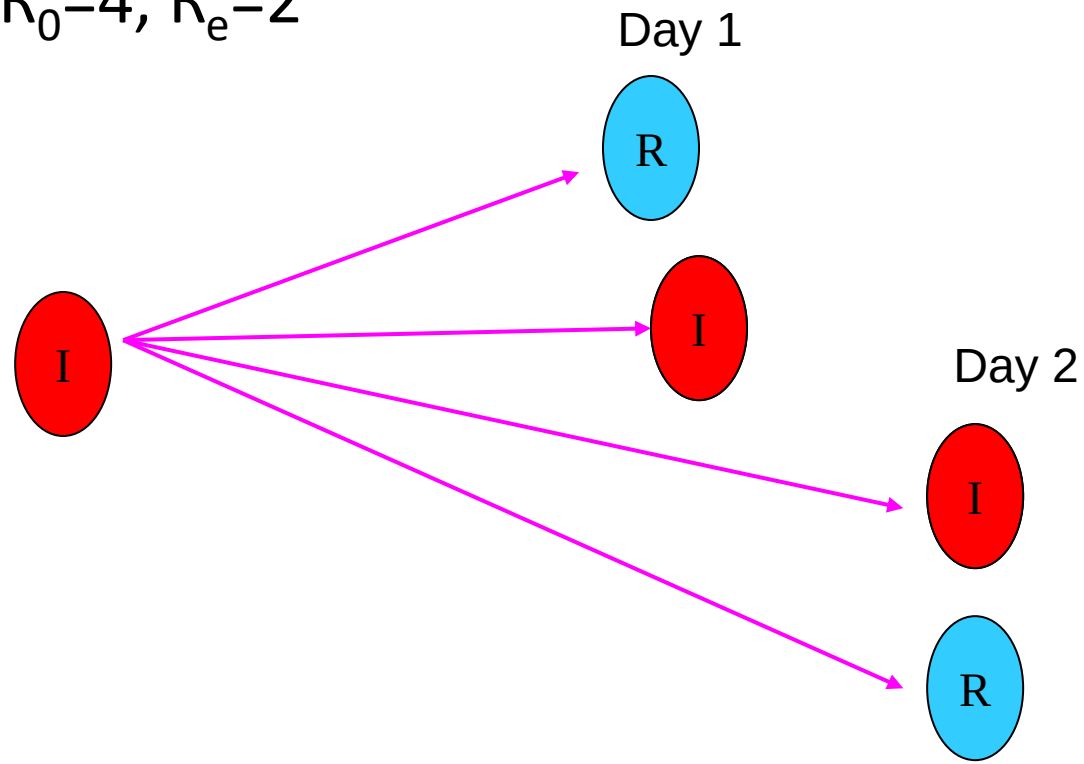
SOCIAL DISTANCING
&
LOCKDOWNS

$R_0=4$



4 days later

$R_0=4$, $R_e=2$



R0

The number of people who develop an infectious disease as the result of infection by a single infectious case introduced into an entirely susceptible population.

R_E

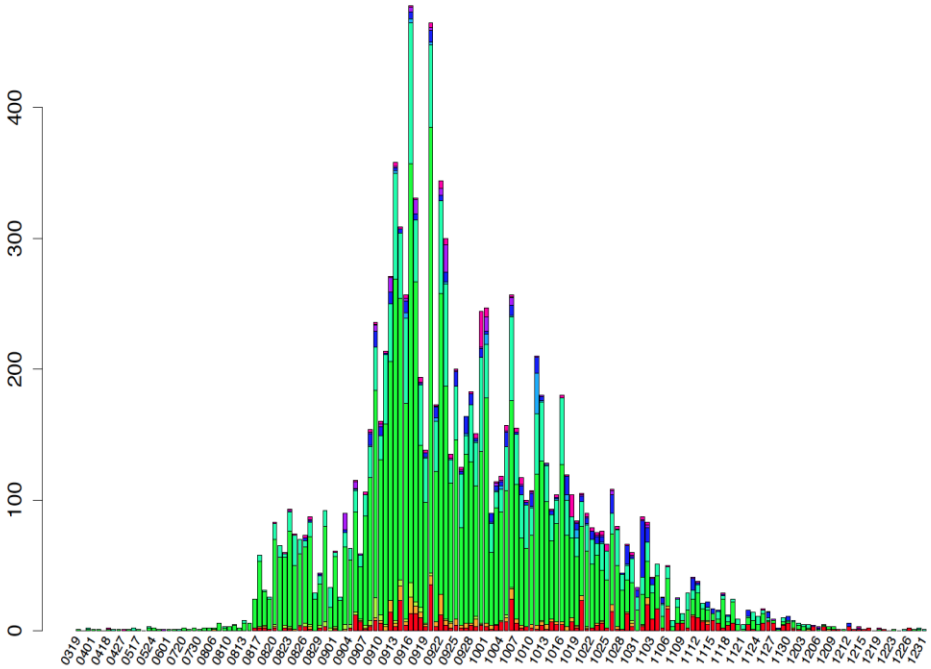
The number of people who develop an infectious disease as the result of infection by a single infectious case introduced into a population of **both susceptible and not susceptible** population.

The effective reproductive number R_E

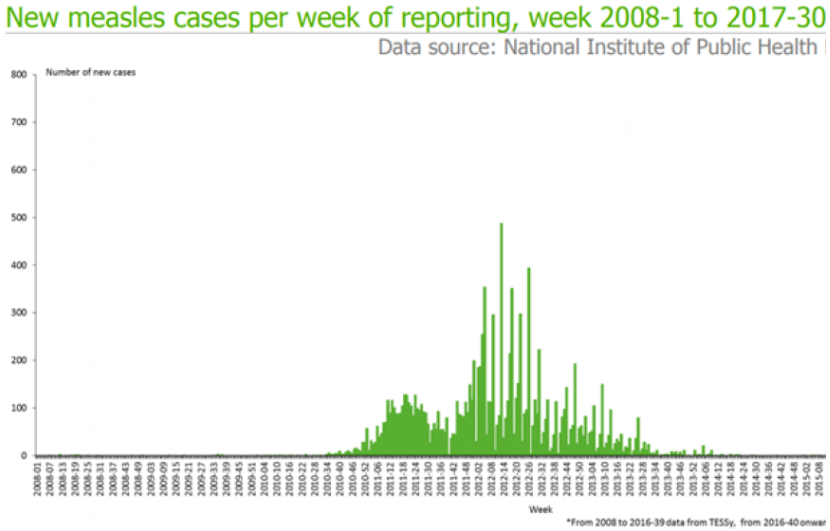
- Population not entirely susceptible
- Fewer people infected by a single case
- $R_E = R_0 * x$

Where x = proportion of contacts in the population that are with susceptible people

Susceptible depletion….

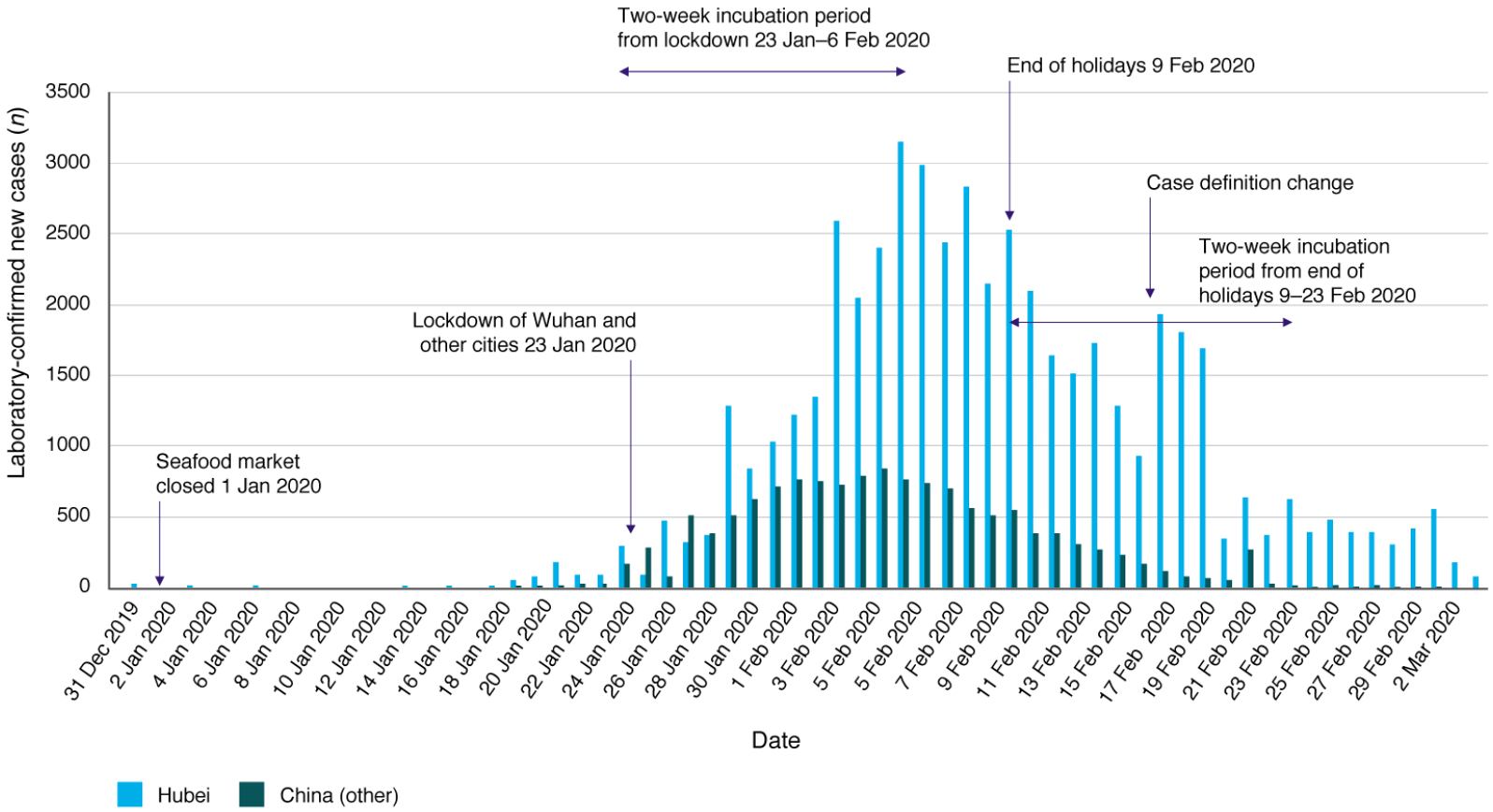


Dengue

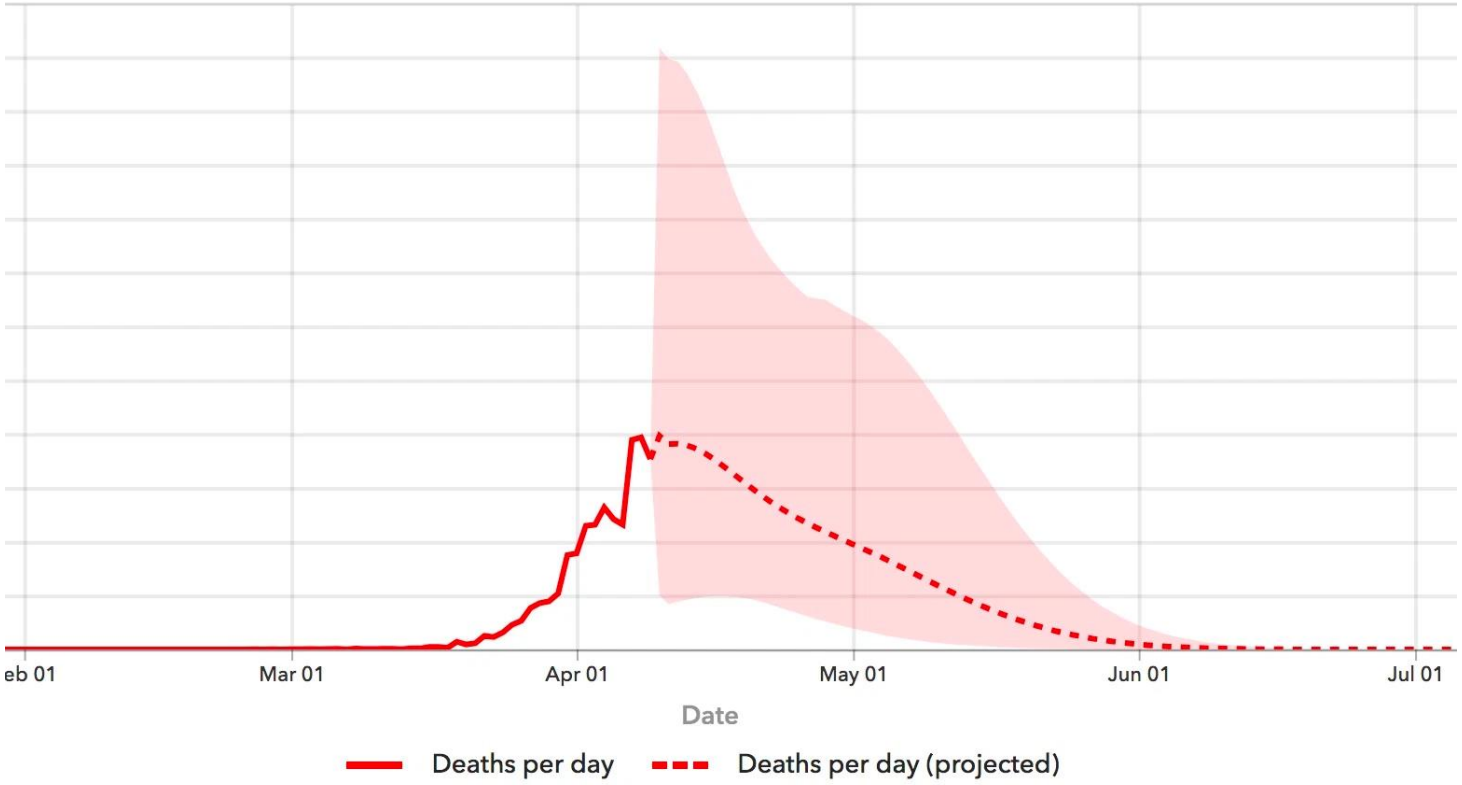


Measles

Susceptible depletion???



IHME model was a **statistical model** assuming same dynamics as Wuhan



Effective reproductive number

- Not a stable attribute of an infectious disease
- Useful if disease is endemic (or at equilibrium) since R_e=1
- If chickenpox is "endemic" and 20% of the population is susceptible
  - $R_e = R_0 x$ and $R_e = 1$
  - R0 = 1/.2 = 5
- Can use serological data to estimate R_0

Mumps model assuming SIR framework: what is R_0 ?

- Mumps model
  - bk = 3
  - D= 1 week (d=1/D)
  - R_0 = bk/d = 3/1 = 3

$$R0 = bk/r$$

Disease	R0
Measles	12-18
Pertussis	12-17
Diphtheria	6-7
Smallpox	5-7
Rubella	5-7
Mumps	4-7
1918 Influenza	1.8-3