Incorporating complexity into the SIR model

MATHEMATICAL MODELING FOR INFECTIOUS DISEASE PLANNING IN AFRICA

(30 minutes)

UNIVERSITY OF Global Health EQUITY
HARVARD T.H. CHAN SCHOOL OF PUBLIC HEALTH
CENTER for COMMUNICABLE DISEASE DYNAMICS

Learning Objectives

1. Understand the limitations of the SIR model
2. Understand how to extend the basic SIR framework to account for complexities in the disease transmission process

What are some limitations of the SIR model we studied?

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- Assumption of infectiousness upon infection does not allow to account for latent infection when modeling diseases with a significant latent period.
- Examples of such diseases: ?
- Implications for TB?

Scenario

There are reports of an outbreak of a new emerging infectious disease that is transmitted from human to human. Not much is known about it, but we've been asked to create some initial models to analyze its spread. Based on initial reports, it appears:

- People have ~20% daily probability of getting infected upon contact with infected individuals
- People recover on average 10 days after infection

BREAKING NEWS:

We are now getting reports that people are not infectious immediately after getting infected. It appears to take about 3 days before people become infectious.

How do we account for a latent period?

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$$S \xrightarrow{\beta} I \xrightarrow{\gamma} R$$

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SLIR model structure:

$$S \xrightarrow{\beta} L \xrightarrow{\alpha} I \xrightarrow{\gamma} R$$

Recall equations for SIR model:

$$\frac{dS}{dt} = \frac{-\beta SI}{N},$$

$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I,$$

$$\frac{dR}{dt} = \gamma I,$$

$$N = S + I + R.$$

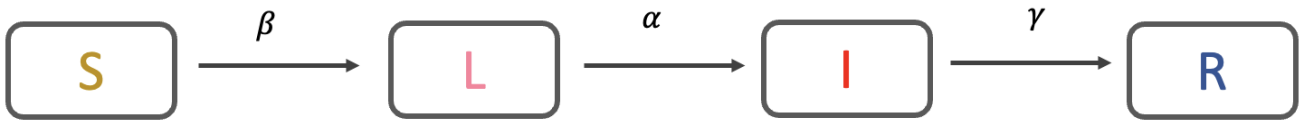
How do we modify the SIR equations to account for latent compartment (L)? Assume **α is the rate of progression from L to I.**

How do we account for a latent period?

Model equations:

α = rate of progression from L to I

= 1/length of latent period



$$\frac{dS}{dt} = \frac{-\beta SI}{N},$$

$$\boxed{\frac{dL}{dt} = \frac{\beta SI}{N} - \alpha L,}$$

$$\frac{dI}{dt} = \alpha L - \gamma \mathrm{I},$$

$$\frac{dR}{dt} = \gamma \mathrm{I},$$

$$N = S + L + I + R.$$

Dynamics of the SLIR model

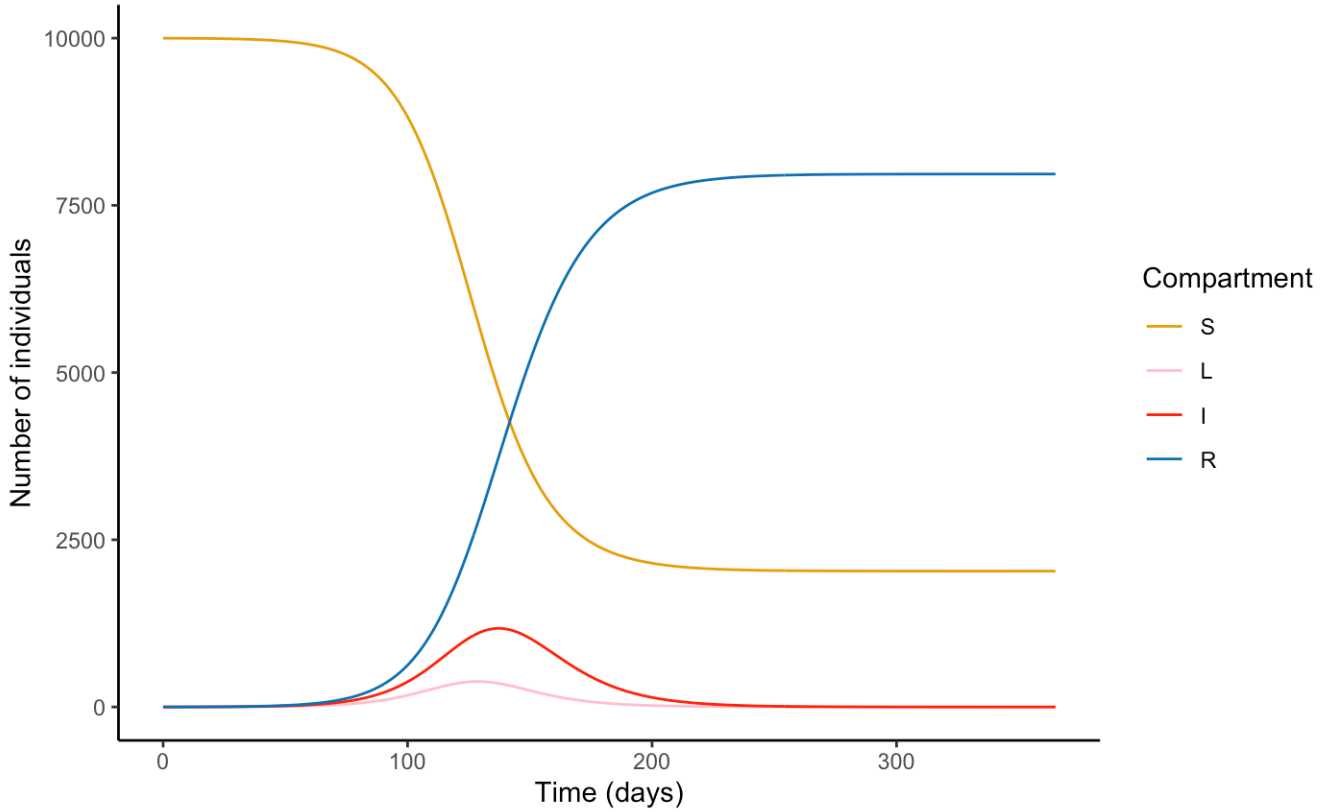


Figure: Dynamics of the number of susceptible (S), latently infected (L), infectious (I) and recovered (R) individuals in a population of 10,000 after the introduction of one infected individual. $S(0) = 9999$, $L(0) = 1,\ I(0) = 1,\ R(0) = 0,\ \beta = 0.2,\ \alpha = 1/3,\ \ \gamma = 1/10.$

What’s next

- **Activity**: Opportunity to apply concepts discussed to COVID-19 and later, your disease of focus
- Explore how dynamics change as you alter parameter values.