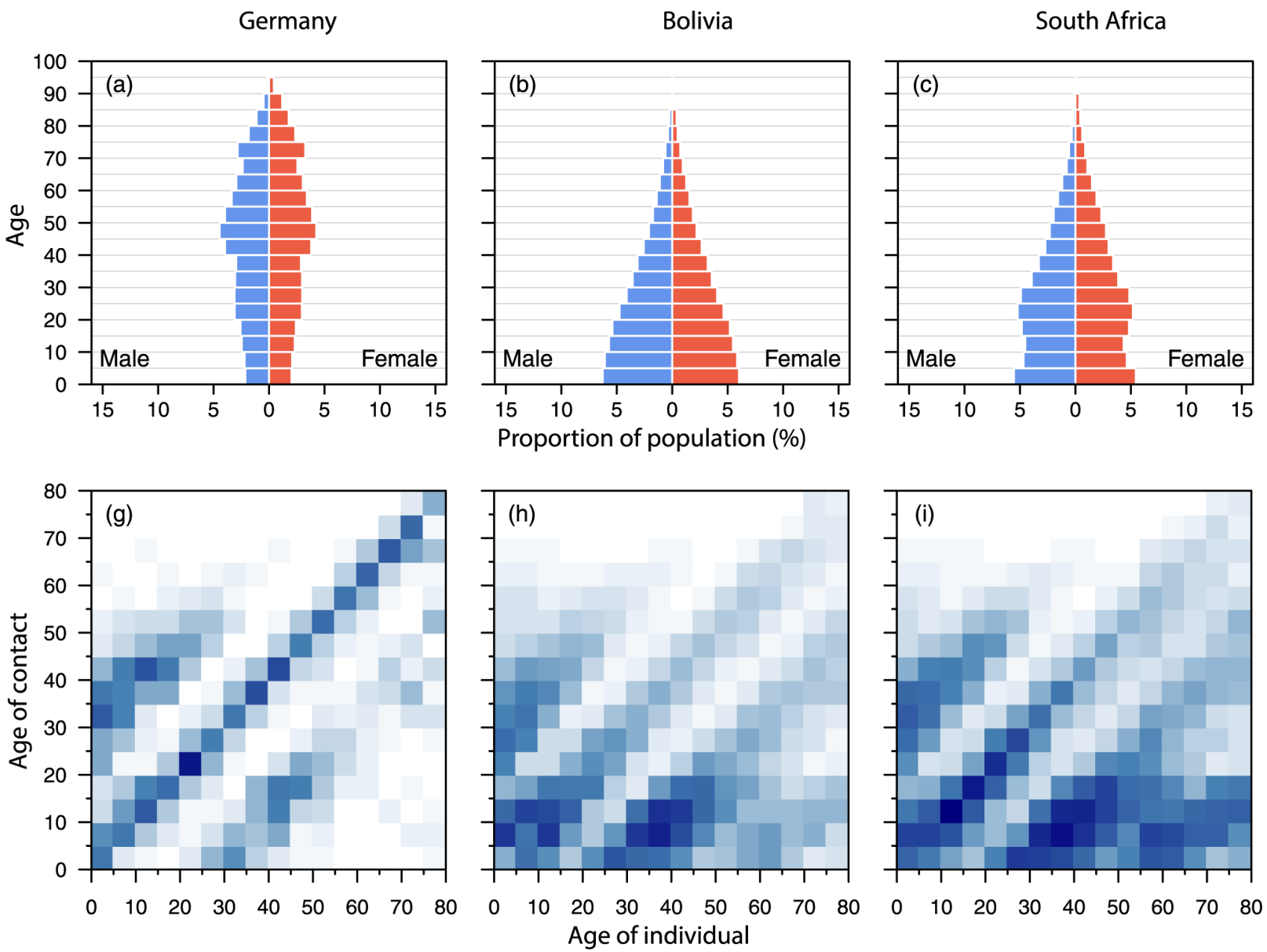
Disease-specific models and heterogeneity

MATHEMATICAL MODELING FOR INFECTIOUS DISEASE PLANNING IN AFRICA

(45 minutes)

Based on last week, I am going to cover the following topics:

- Age stratification
- Water-borne disease transmission
- Vector-borne disease transmission
- Agent based models (sort of)

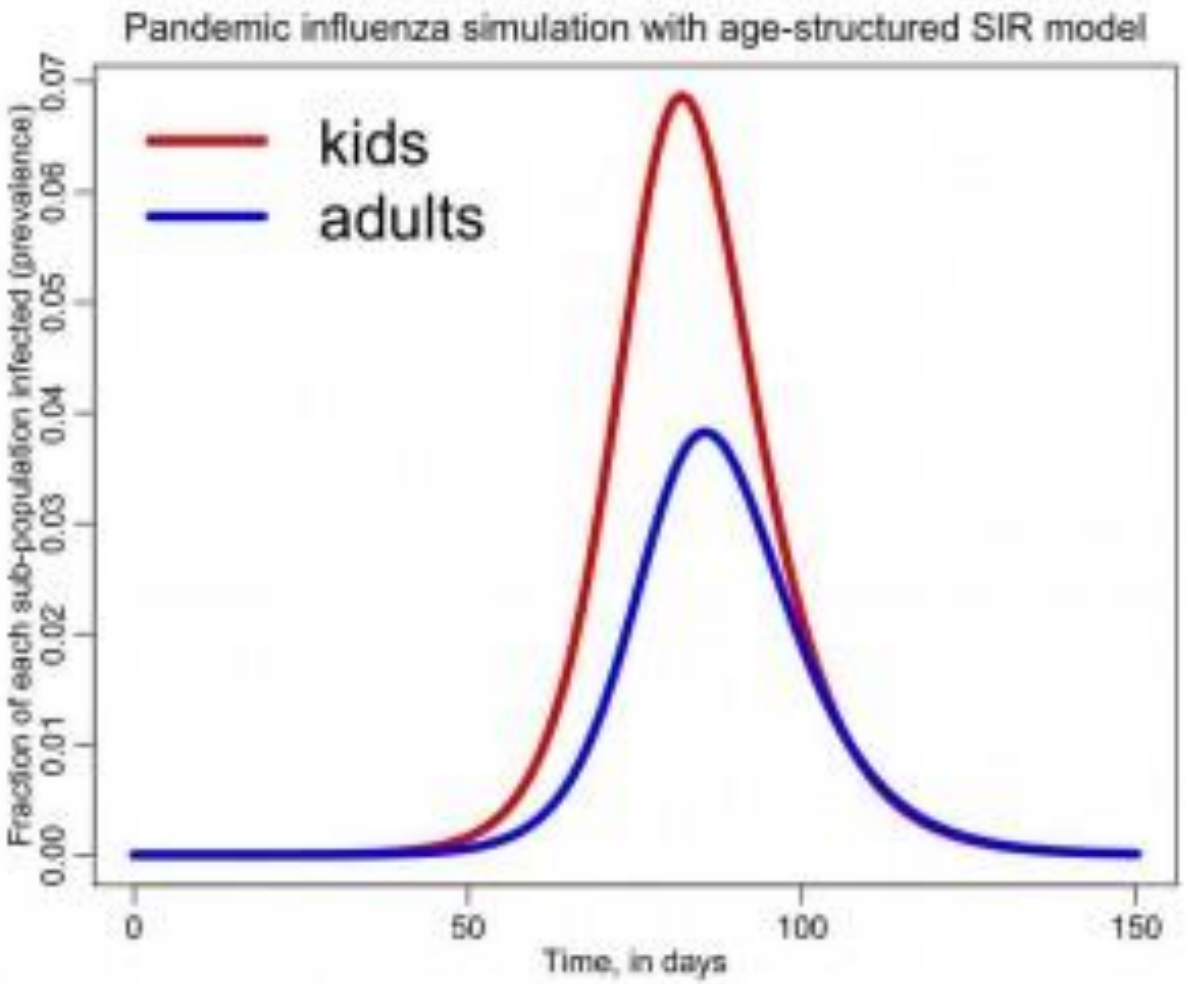


Prem et al (2017) PLoS

$$
\begin{aligned}
\frac{dS_i}{dt} &= -\beta S_i \sum_j C_{ij} I_j / N_j \\
\frac{dI_i}{dt} &= +\beta S_i \sum_j C_{ij} I_j / N_j - \gamma I_i \\
\frac{dR_i}{dt} &= +\gamma I_i
\end{aligned}
$$

$$
C = \begin{pmatrix} 18 & 9 \\ 3 & 12 \end{pmatrix}
$$

where i and j are the indices of the age classes, and N_j is the population in age group j.



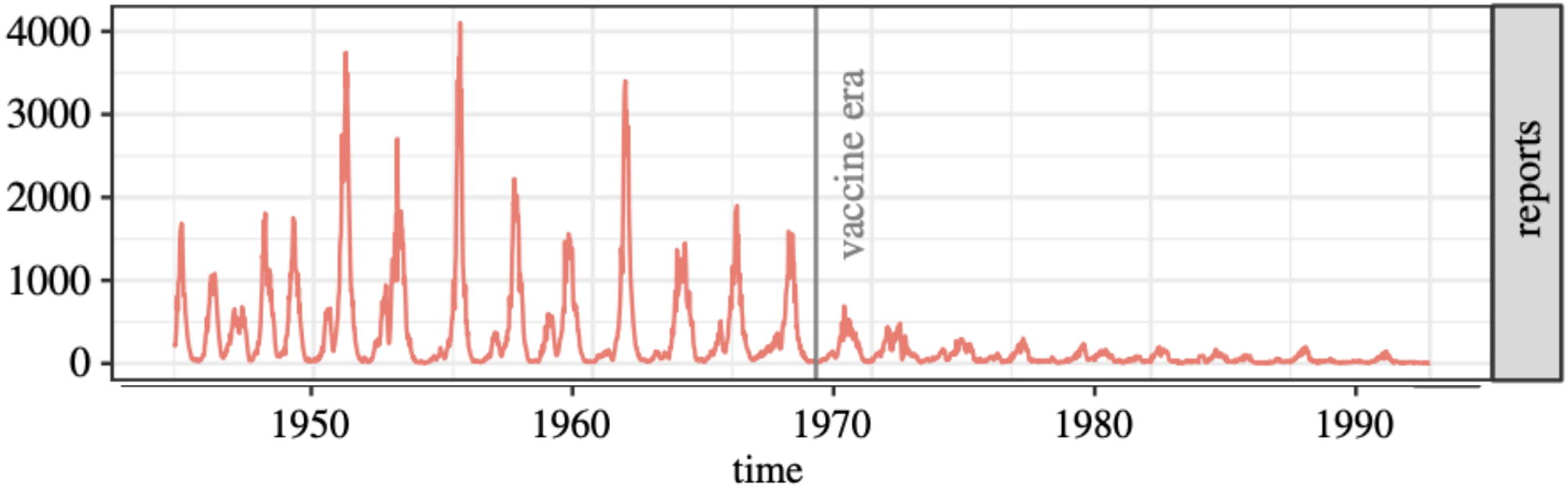
$$C = \begin{pmatrix} 18 & 9 \\ 3 & 12 \end{pmatrix}$$

Age-structure and transient dynamics in epidemiological systems

F. M. G. Magpantay[1], A. A. King[2] and P. Rohani[3]

Cite this article: Magpantay FMG, King AA, Rohani P. 2019 Age-structure and transient dynamics in epidemiological systems. *J. R. Soc. Interface* **16**: 20190151.
http://dx.doi.org/10.1098/rsif.2019.0151

Measles in England and Wales



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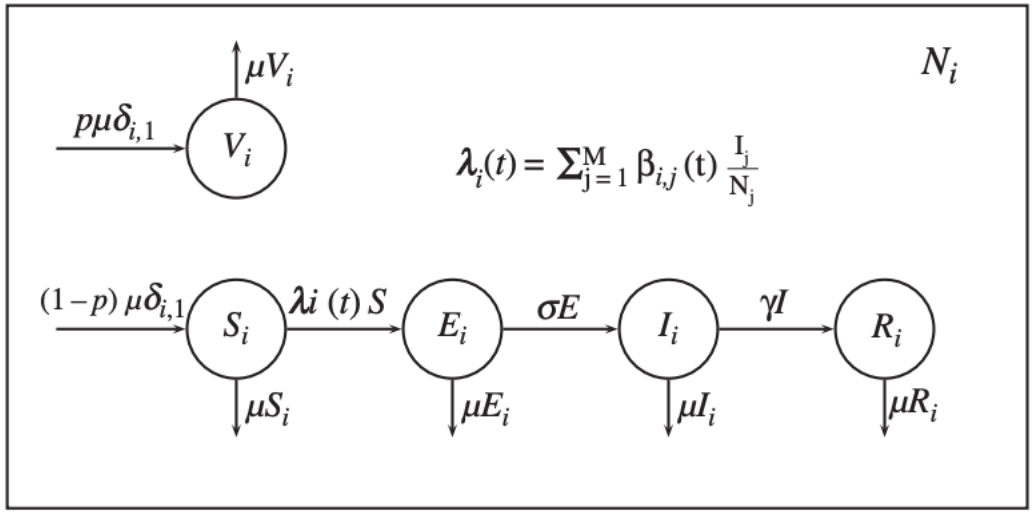

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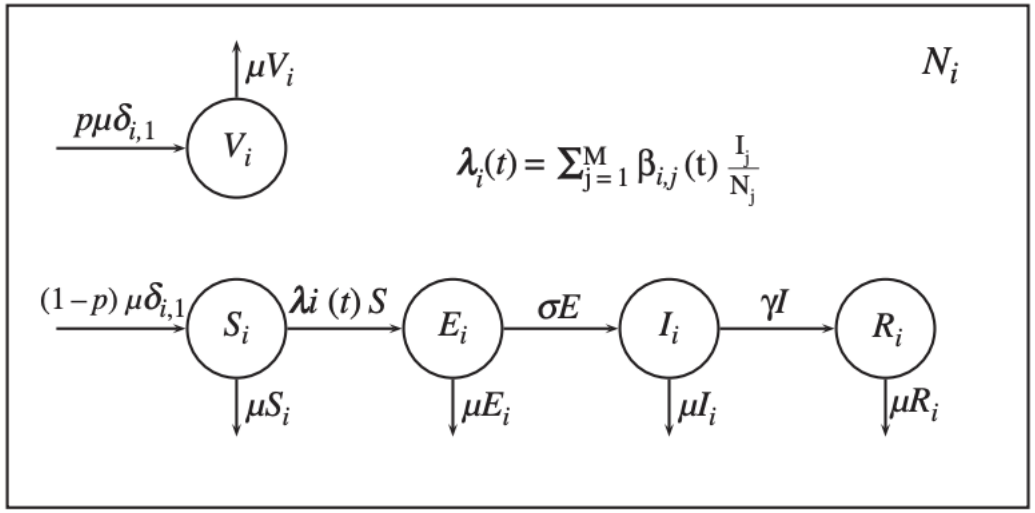
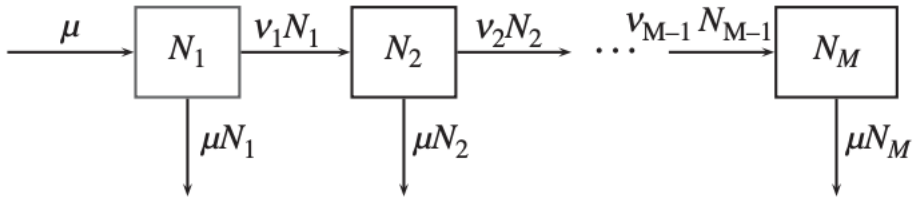
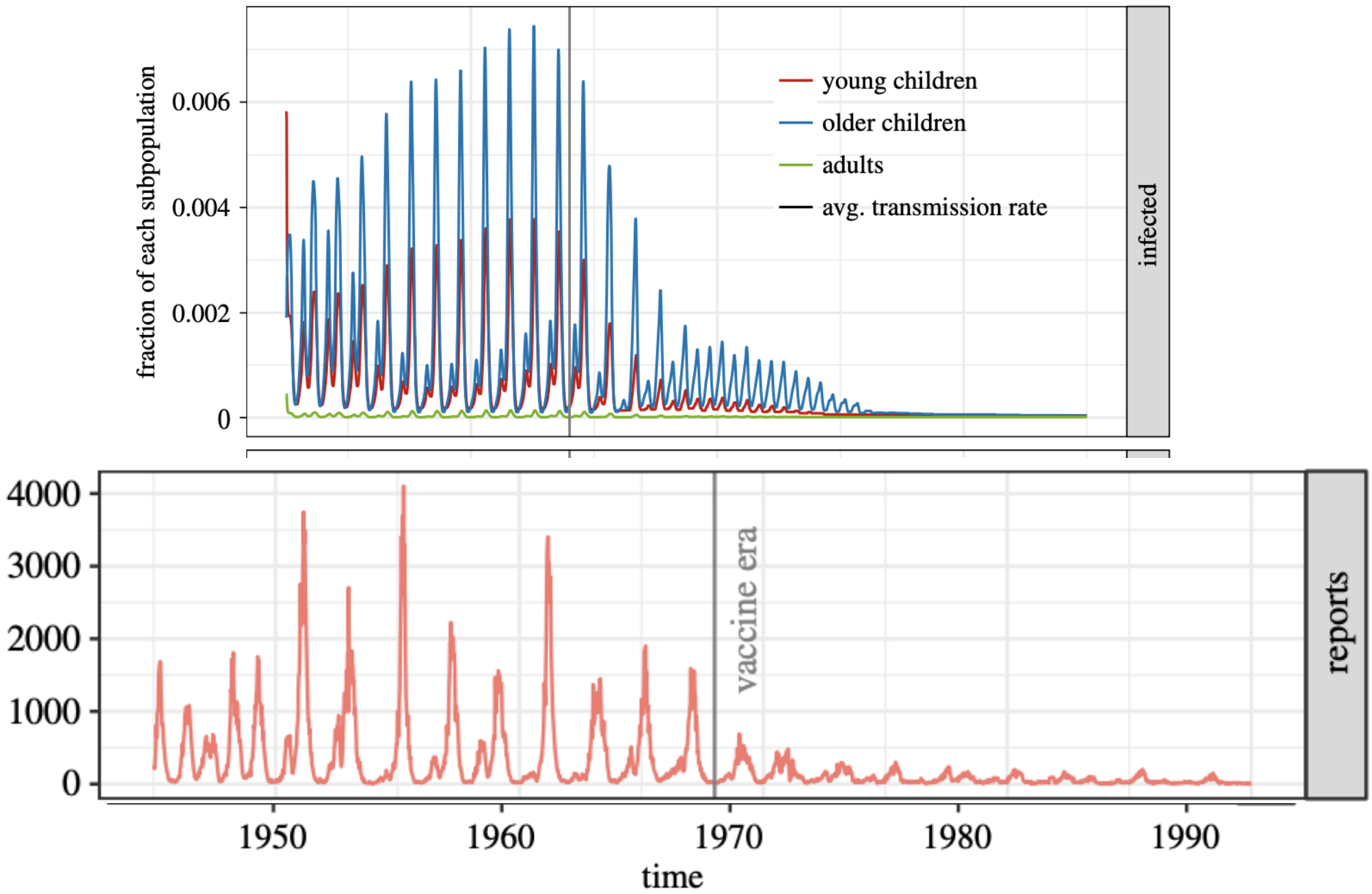




For $i = 1, \ldots, M$,

$$\left.\begin{aligned}
\frac{dV_i}{dt} &= p\mu\delta_{i,1} - \mu V_i + \nu_{i-1}V_{i-1} - \nu_i V_i,\\
\frac{dS_i}{dt} &= (1-p)\,\mu\delta_{i,1} - \lambda_i S_i - \mu S_i + \nu_{i-1}S_{i-1} - \nu_i S_i,\\
\frac{dE_i}{dt} &= \lambda_i S_i - \sigma E_i - \mu E_i + \nu_{i-1}E_{i-1} - \nu_i E_i,\\
\frac{dI_i}{dt} &= \sigma E_i - \gamma I_i - \mu I_i + \nu_{i-1}I_{i-1} - \nu_i I_i\\
\text{and}\quad \frac{dR_i}{dt} &= \gamma I_i - \mu R_i + \nu_{i-1}R_{i-1} - \nu_i R_i.
\end{aligned}\right\} \tag{2.1}$$

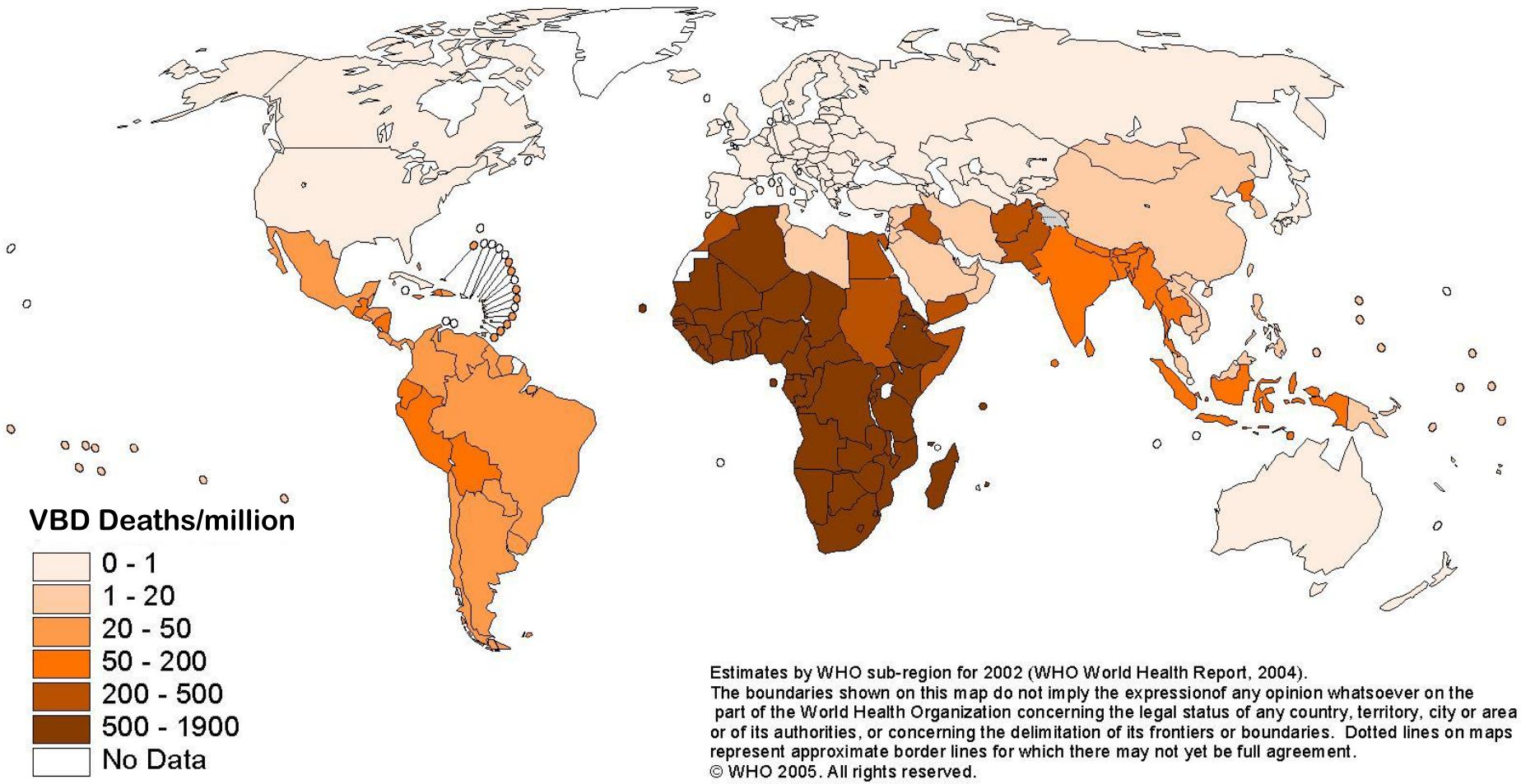
Kronecker delta, which equals one if $i = j$ and zero otherwise


fraction of each subpopulation
0.006
0.004
0.002
0
young children
older children
adults
avg. transmission rate
infected
4000
3000
2000
1000
0
vaccine era
reports
1950
1960
1970
1980
1990
time

Non-direct transmission

- Estimating “contact” (human -> X -> human) parameters difficult when environmental/ecological variables are involved
- Heterogeneities in these variables are very large in some cases (and hard to measure)
- R0 can vary widely in time and space and may be much larger than for directly transmitted infections

Deaths from vector-borne disease



Estimates by WHO sub-region for 2002 (WHO World Health Report, 2004).
The boundaries shown on this map do not imply the expressionof any opinion whatsoever on the part of the World Health Organization concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. Dotted lines on maps represent approximate border lines for which there may not yet be full agreement.
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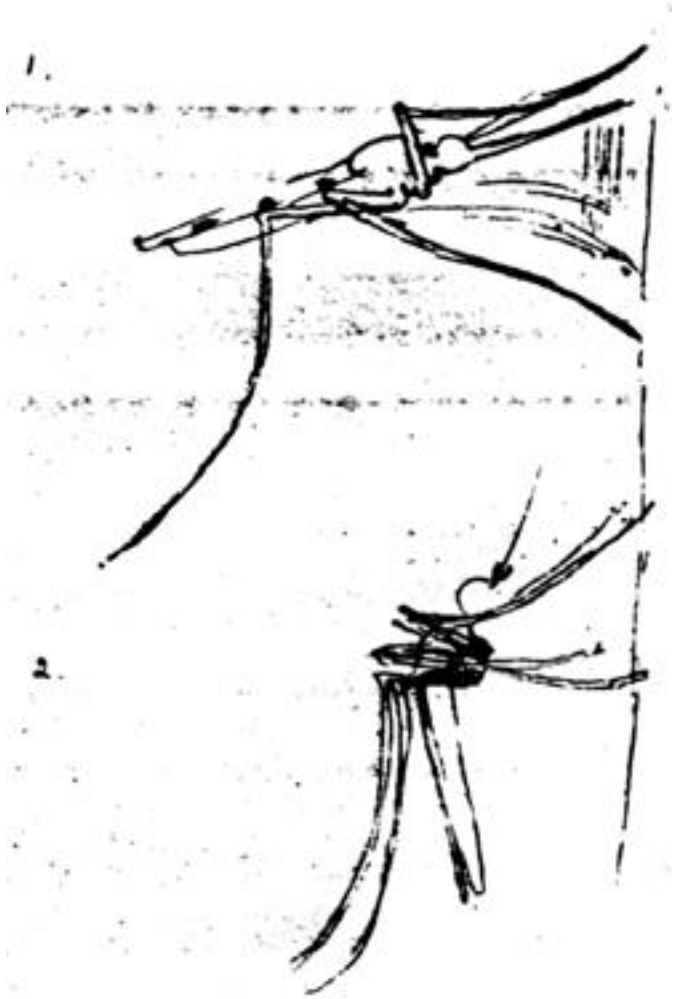
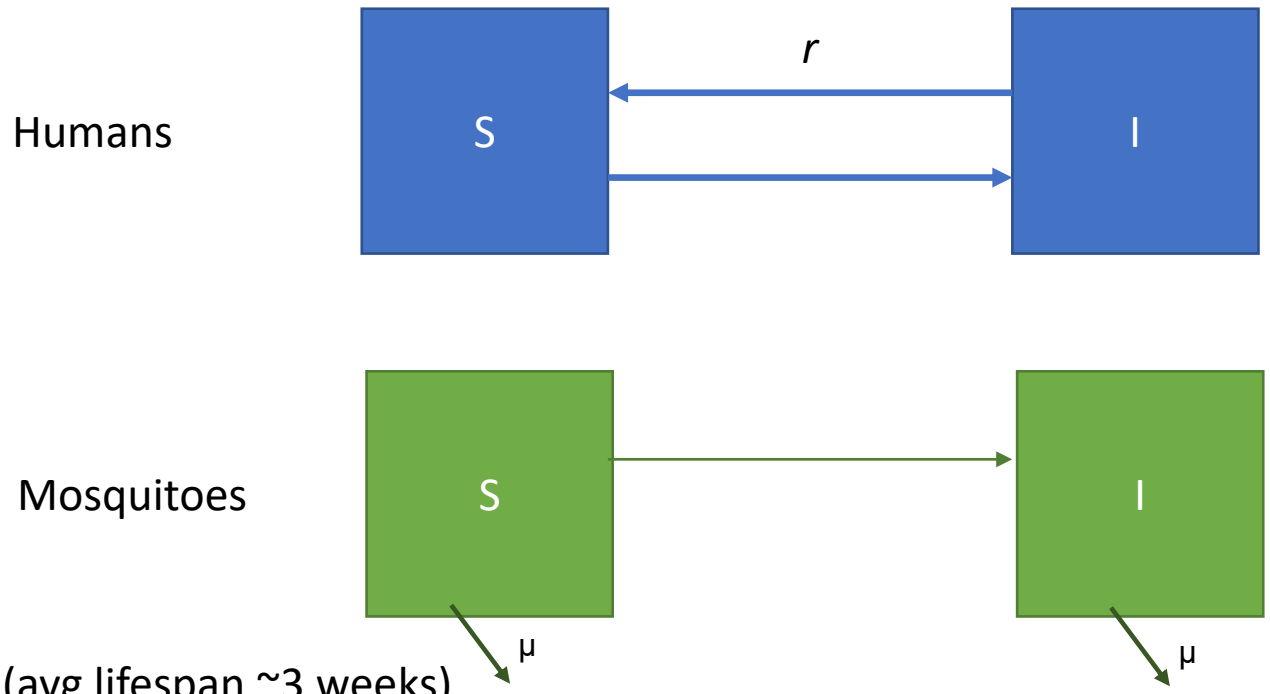
Ronald Ross, 1897



http://www.cdc.gov/Malaria/history/ross.htm

What would a two-species model look like?

- Assume that both the humans and the mosquitoes can be modeled as an S->I model (no immunity, but mosquitoes don't live long enough to lose infection)



Mosquito death rate (avg lifespan ~3 weeks)

Connected by the **biting rate**:

Mosquitoes become infected based on how many bites are on infected people and how infectious infected people are per infectious bite

(note: how is this similar to our transmission coefficient depending on *k* and *b*)

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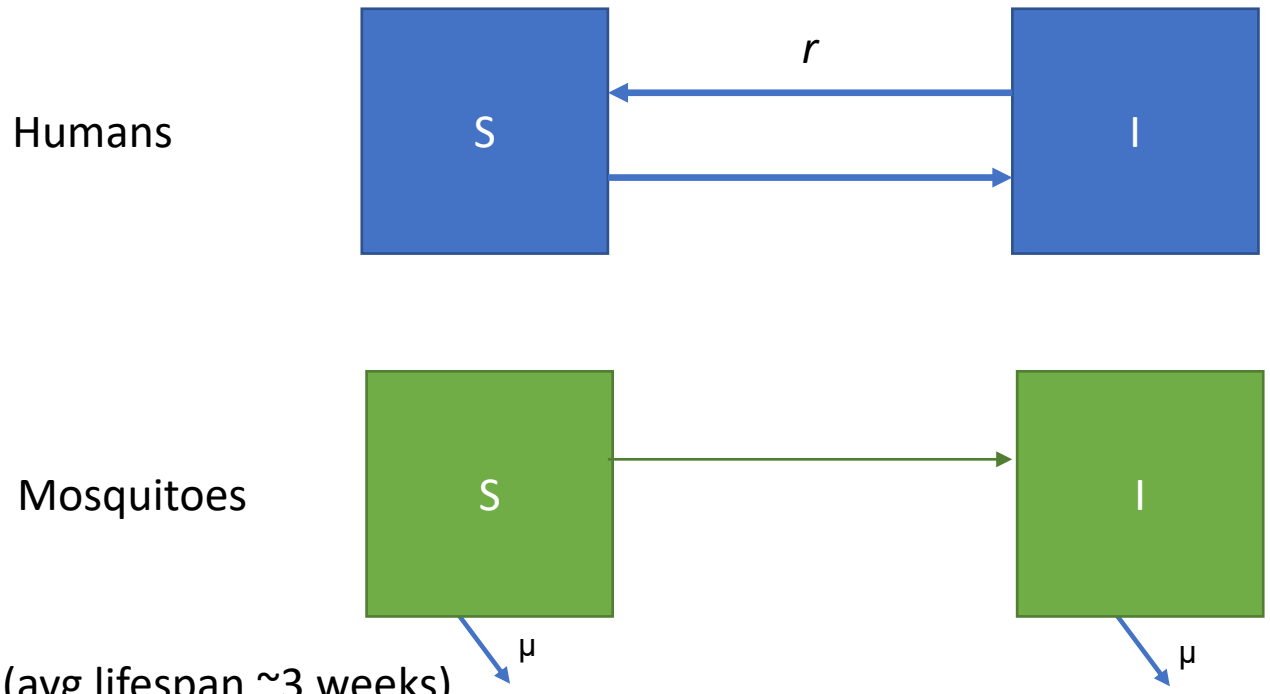

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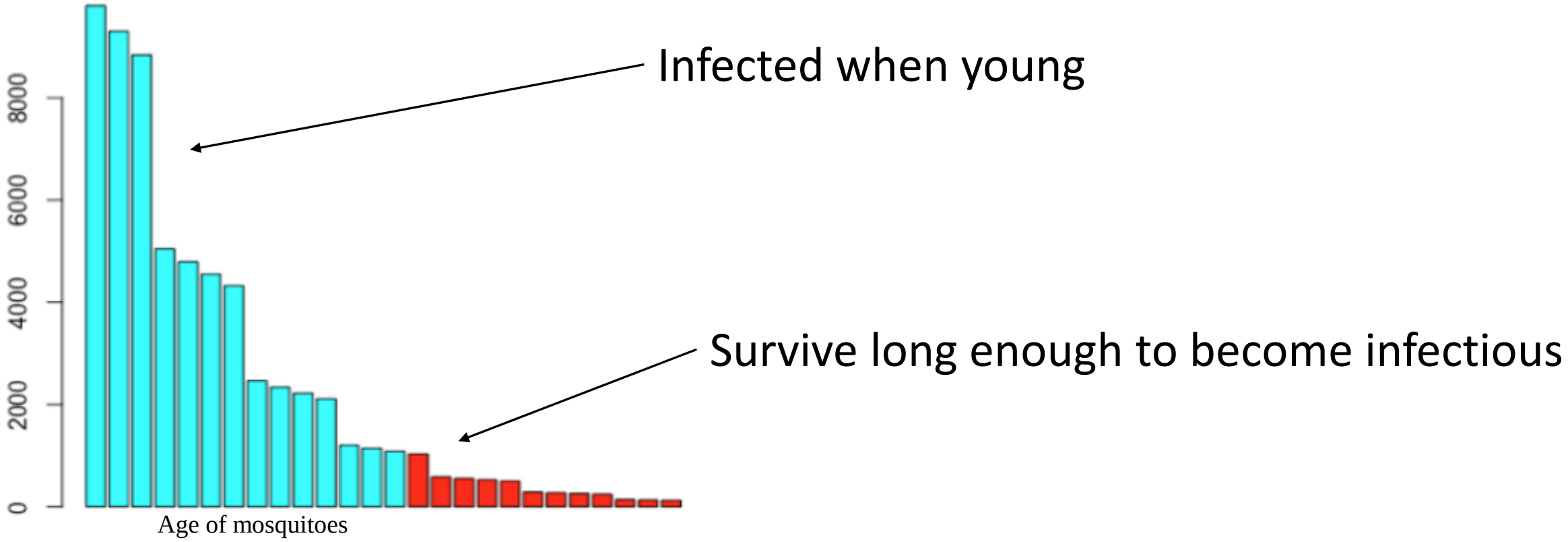
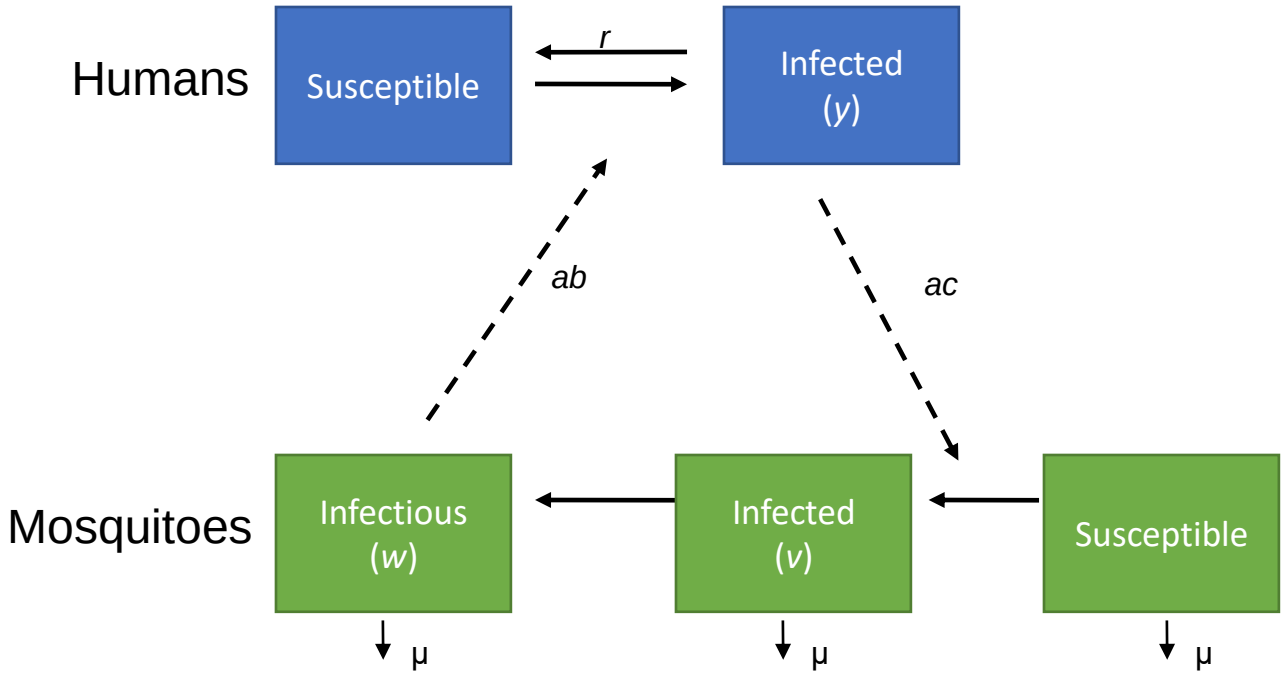
Distribution of ages of mosquitoes in a population



Ross-MacDonald model
(1911,1957)



$$\frac{dy}{dt} = mabw(1-y) - ry \qquad (1)$$

$$\frac{dv}{dt} = acy(1-v-w) - acy(1-v-w)e^{-\mu T} - \mu v \qquad (2)$$

$$\frac{dw}{dt} = acy(1-v-w)e^{-\mu T} - \mu w \qquad (3)$$

Mosquito feeding rate

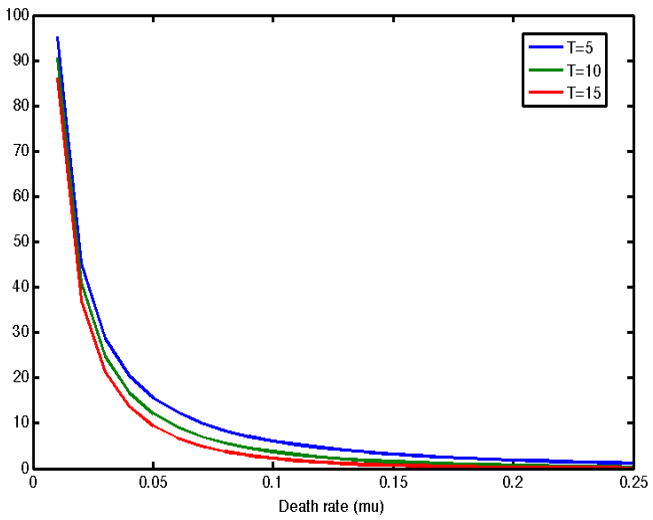
Infectiousnes

Ratio mosquito:human

$$R_0 = \frac{ma^2bce^{-m\tau}}{rm}$$

recovery

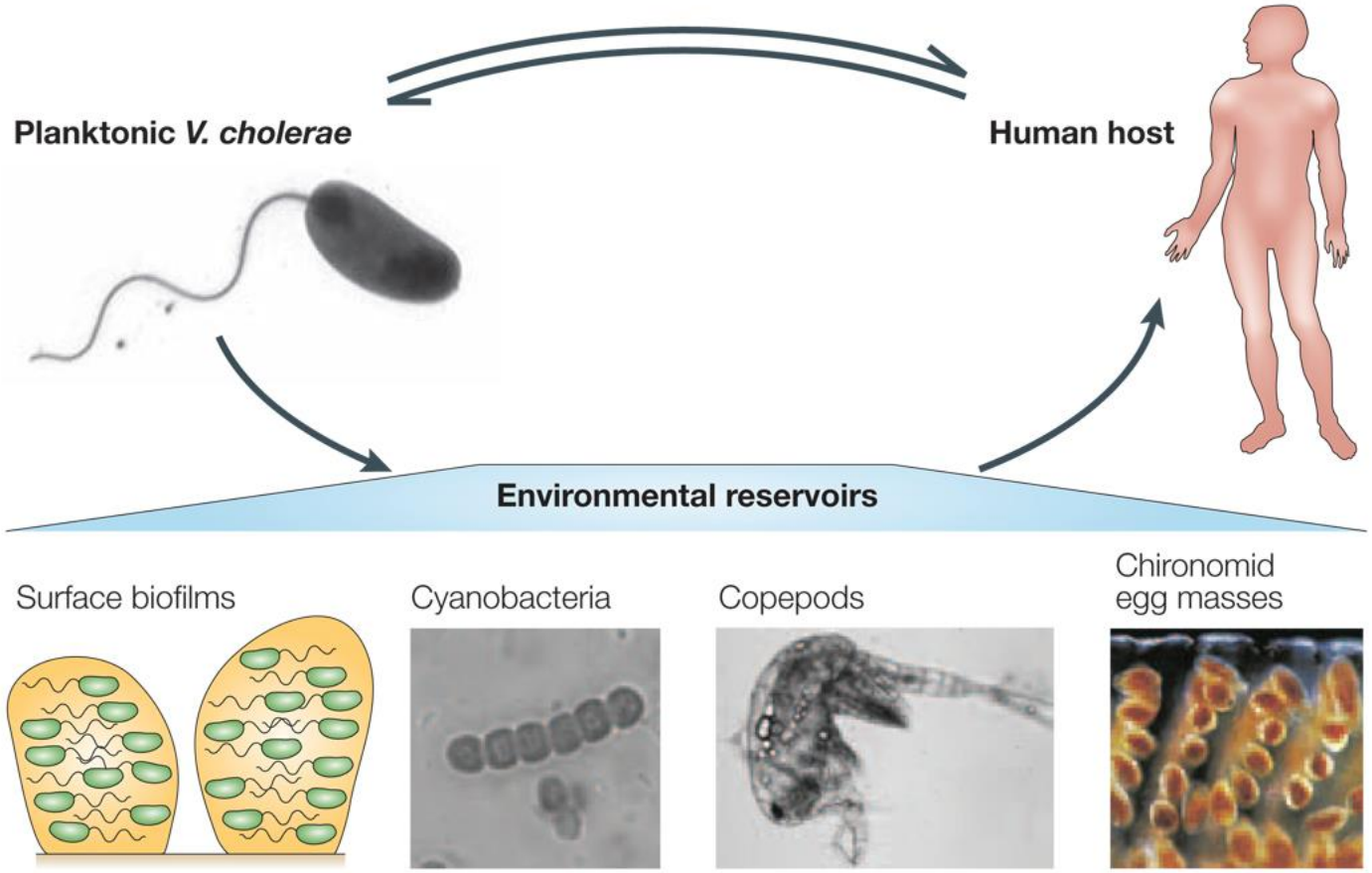
Mosquito death



The probability that a mosquito survives to become infectious

1. Biting rate squared (two bites per transmission)
2. Exponential relationship between R0 and μ, latent period close to lifespan of mosquito

Cholera transmission
(through the environment)



Butler & Camilli, 2005, Nat Rev Microbiol

Cholera transmission
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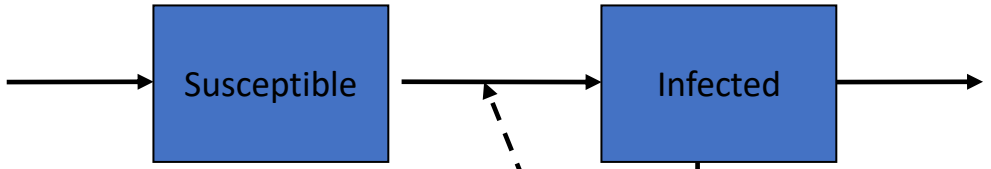
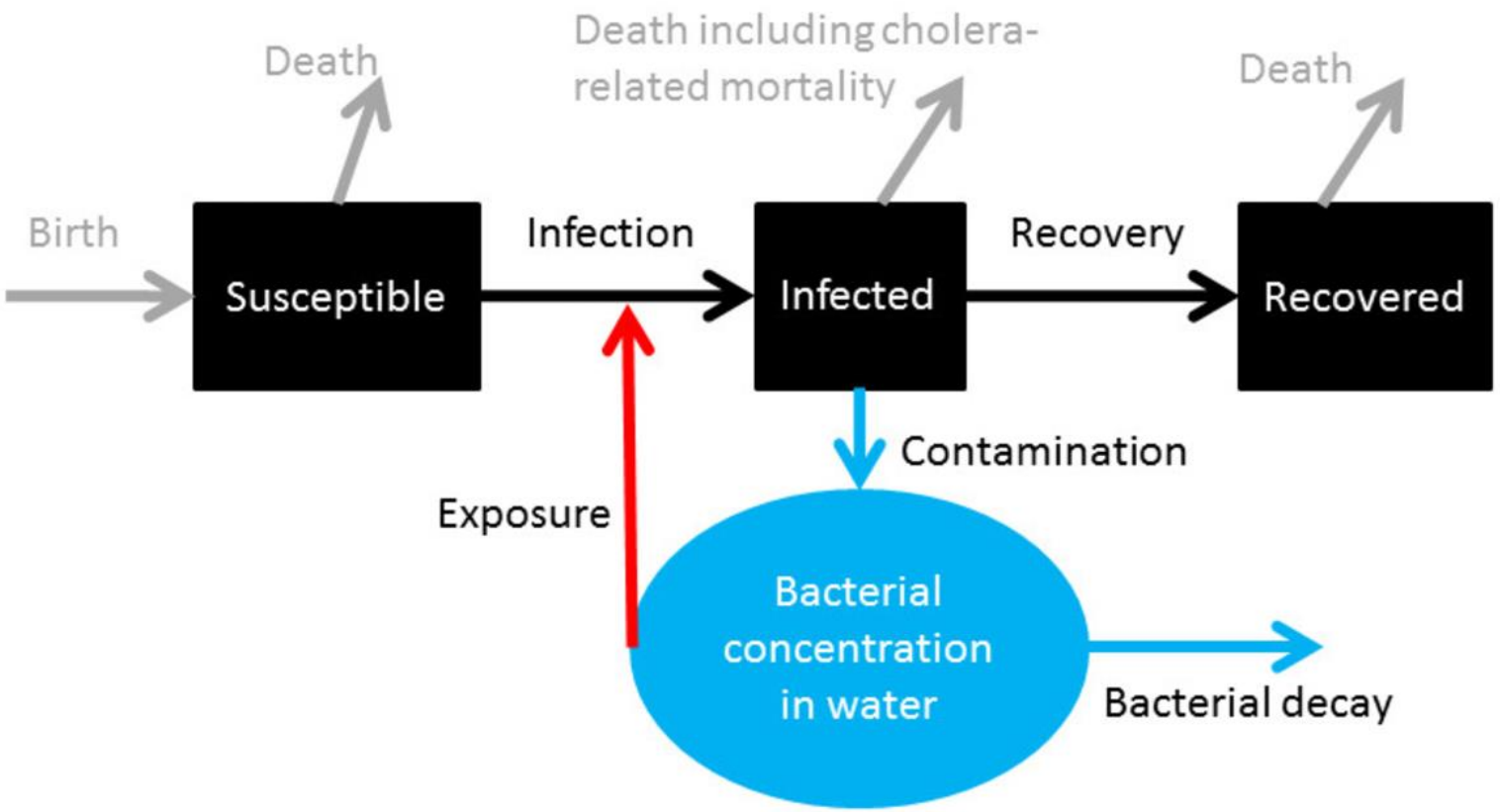




Fung 2014 Emerging Themes in Epi

Cholera transmission

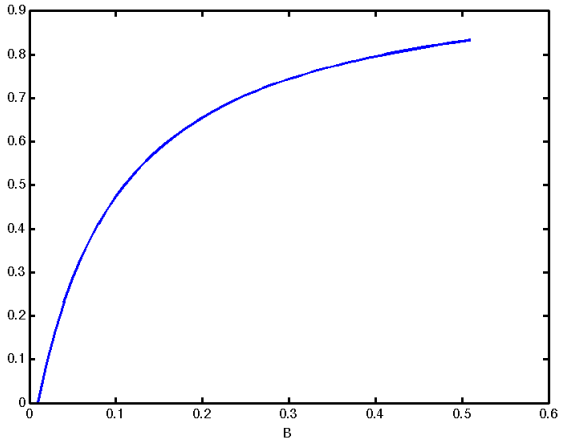
adapted from Codeco (2001) BMC Inf Dis

$$\frac{dS}{dt} = \mu N - \beta \frac{B}{B+\kappa} S$$

Births

"Contact rate"/day

Logistic relationship between contribution to infective water in cells/mL and infective "concentration"



Courtesy of Yonatan Grad

Cholera transmission

$$\frac{dS}{dt} = \mu N - \beta \frac{B}{B+\kappa} S$$

$$\frac{dI}{dt} = \beta \frac{B}{B+\kappa} S - \gamma I$$

Recovery rate

Cholera transmission

$$\frac{dS}{dt} = \mu N - \beta \frac{B}{B+\kappa} S$$

$$\frac{dI}{dt} = \beta \frac{B}{B+\kappa} S - \gamma I$$

$$\frac{dB}{dt} = \xi I - \delta B$$

Half-life of *Vibrio cholerae* in the water supply

Contribution to contaminated water cells·ml^{-1}·day^{-1}·person^{-1}

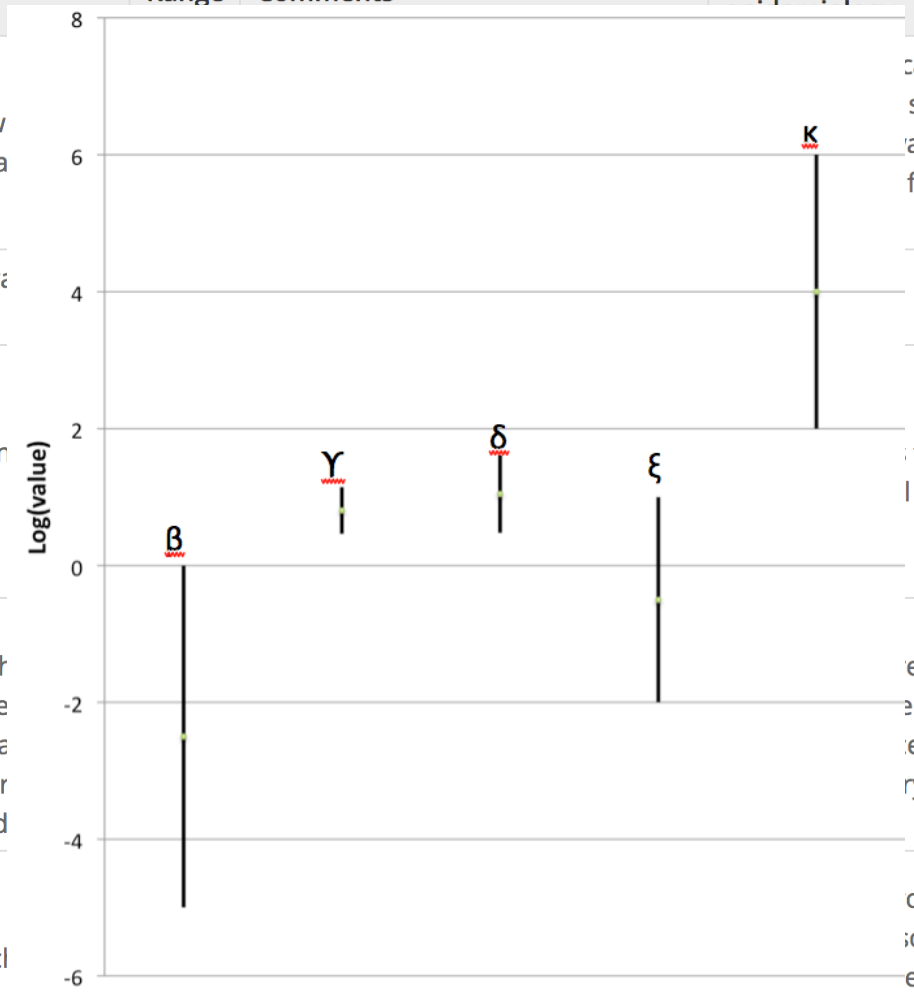
$$\frac{dS}{dt} = \mu N - \beta \frac{B}{B+\kappa} S$$

$$\frac{dI}{dt} = \beta \frac{B}{B+\kappa} S - \gamma I$$

$$\frac{dB}{dt} = \xi I - \delta B$$

$$R_0 = N \frac{bx}{kgd} \gg 3$$

Symbol	Parameters	Range	Comments	Potential data from field
β	Rate of "contact" w reservoir water (da			cation of sources; ater usage and from these
1/γ	Duration of cholera infection (days)			
1/δ	Cholera life span in reservoir (days)			for l experiments
ξ	Rate of water contamination by h i.e. rate of increase *cholerae* concentra the water reservoir mL^{-1} * $person^{-1}$ * d			equency and ery stool and entration of ry stool.
κ	Concentration of cl that yields 50% chance of infection (cells/mL)	10^6	data provided data for doses (number of bacteria), the parameter measures in concentration.	olume of water son per day and entration in the water samples, one can estimate the dose of vibrio intake per person per day



Courtesy of Yonatan Grad

Additional complication: spatial heterogeneity

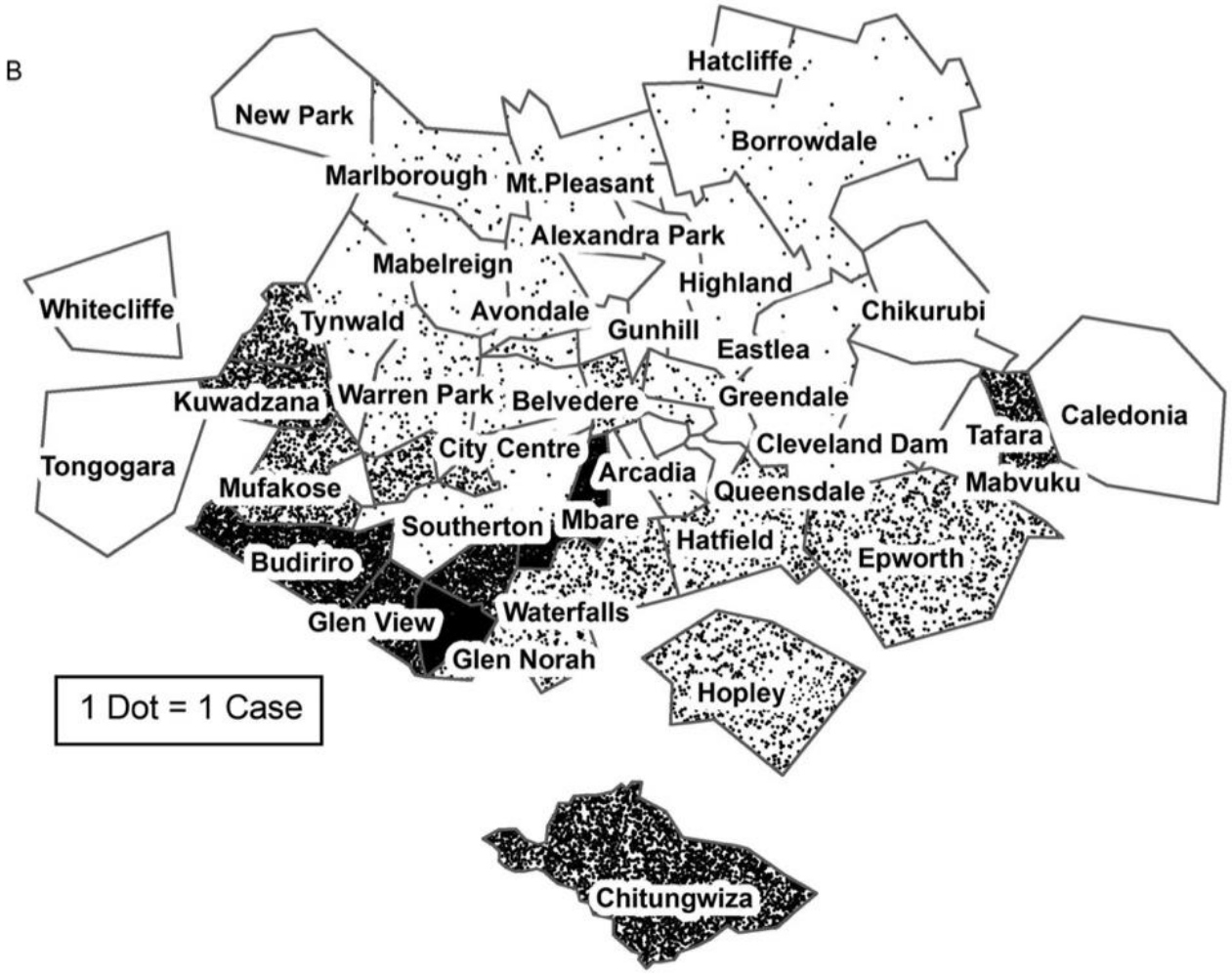


Figure 2. Map of greater Harare, including administrative boundaries, industrial areas, railway, bus stops and markets (2A) in relation to the distribution of cholera cases by suburbs (2B), 2008-2009 ($n = 19\,422$ cases).

Luque Fernandez *et al.*, Trans. Royal Soc. Trop Med & Hygiene, 2010

(a) **Ross model** [36] $\frac{dI_h}{dt} = a\,b\,m\,I_m\,(1 - I_h) - r\,I_h$ $\frac{dI_m}{dt} = a\,c\,I_h\,(1 - I_m) - \mu_2\,I_m$	$\frac{m a^2 b c}{r \mu_2}$	a : Man biting rate [0.01-0.5 day^{-1}] b : Proportion of bites that produce infection in human [0.2-0.5]
(b) **Macdonald model** [40] $\frac{dI_h}{dt} = a\,b\,m\,I_m(1 - I_h) - r\,I_h$ $\frac{dE_m}{dt} = a\,c\,I_h(1 - E_m - I_m) - a\,c\,I_h(t - \tau_m)\left[1 - E_m(t - \tau_m) - I_m(t - \tau_m)\right] e^{-\mu_2 \tau_m} - \mu_2 E_m$ $\frac{dI_m}{dt} = a\,c\,I_h(t - \tau_m)\left[1 - E_m(t - \tau_m) - I_m(t - \tau_m)\right] e^{-\mu_2 \tau_m} - \mu_2 I_m$	$\frac{m a^2 b c}{r \mu_2} e^{-\mu_2 \tau_m}$	c : Proportion of bites by which one susceptible mosquito becomes infected [0.5] m : Ratio of number of female mosquitoes to that of humans [0.5-40] r : Average recovery rate of human [0.005-0.05 day^{-1}]